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Analysis of frame structure vibrations induced by traffic

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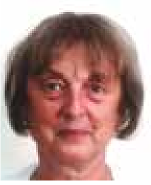

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Preliminary note

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Analysis of frame structure vibrations induced by traffic

A simple numerical model for the dynamic analysis of 2D frames in the frequency domain, based on the Spectral Element Method (SEM), is presented in the paper. The influence of soil-structure interaction is taken into account. The dynamic stiffness of rigid foundations is determined by the Integral Transform Method (ITM). Three frames with a different number of storeys are analyzed with respect to vibrations caused by tram and road traffic. The influence of the soil-structure interaction on natural frequencies and amplitudes of vibrations is considered. The assessment of the way in which humans are affected by traffic-induced vibrations is conducted according to British Standard BS:6472.

Key words:

traffic induced vibrations, frame structure, integral transform method, soil-structure interaction

Prethodno priopćenje

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Analiza vibracija okvirnih konstrukcija uzrokovanih prometom

U radu je prikazan jednostavan numerički model za proračun vibracija ravninske okvirne konstrukcije u frekvencijskoj domeni, primjenom spektralnih elemenata u kome je međudjelovanje tla i zgrade uzeto u obzir. Dinamička krutost krutog temelja određena je metodom integralne transformacije. Analizirane su tri okvirne konstrukcije različite katnosti na djelovanje vibracija od tramvajskog i cestovnog prometa. Razmatran je utjecaj tla na vlastite frekvencije i odgovor konstrukcije. Analiza djelovanje vibracija na ljude provedena je prema britanskom standardu BS:6472.

Ključne riječi:

vibracije od prometa, okvirna konstrukcija, metoda integralne transformacije, međudjelovanje tla i građevine

Vorherige Mitteilung

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Analyse verkehrsbedingter Vibrationen von Rahmenkonstruktionen

In der vorliegenden Arbeit ist ein einfaches, auf der Spektrale-Elemente-Methode (SEM) beruhendes, numerisches Model für die dynamische Analyse von 2D Rahmen in der Frequenzdomäne dargestellt, das die Boden-Bauwerk-Interaktion berücksichtigt. Die dynamische Steifigkeit des Bodens ist mittels der Integraltransformationsmethode (ITM) berechnet. Drei Rahmenkonstruktionen verschiedener Anzahl von Stockwerken sind unter dem Einfluss verkehrsbedingter, durch Straßen- und Straßenbahnverkehr verursachter Vibrationen analysiert. Dabei sind der Einfluss von Boden-Bauwerk-Interaktion auf die Modalfrequenzen und Vibrationsamplituden. Eine Bewertung der verkehrsbedingten Vibrationen in Bezug auf den Menschen, nach den Anordnungen des British Standard BS:6472, ist dargestellt.

Schlüsselwörter:

verkehrsbedingte Vibrationen, Rahmenkonstruktion, Integraltransformationsmethode, Boden-Bauwerk-Interaktion

1. Introduction

The population growth and rapid development of cities has lead to a substantial increase of traffic in urban areas. Significant traffic vibrations are mainly caused by heavy vehicles, i.e. buses, trucks and trams. Traffic vibrations are low frequency disturbances caused by dynamic and oscillation forces of moving wheels. Disturbances propagate through soil in all directions, pass through building foundations, and cause vibrations of buildings, which might negatively affect not only buildings, but also humans or sensitive equipment. The generation of traffic-induced ground vibrations, the transmission path, the effects on humans and their health, the effects on buildings, as well as measures for reducing such vibrations, have been investigated by researchers in many countries, [1, 2]. The need to predict traffic vibration effects on buildings has led to the creation of numerous numerical models, [3, 4]. A comprehensive numerical model, which includes wave propagation through soil, from the source to the building, and building vibrations, i.e. the *source-receiver model,* is a complex model based on general dynamic soil-structure interaction methods [5]. The detailed dynamic analysis is carried out using the finite element method (FEM) in the frequency domain, or the boundary element method combined with the finite element method [3]. However, these methods are not simple and their application is burdened with some numerical problems. On the other hand, current commercial programs are unable to grasp all aspects of this problem. The analysis of traffic vibration effects on reinforced plane frames (RC), based on the commercial program ETABS, is presented in paper [4]. But, the most important factors in traffic-induced building vibrations are not taken into account: the effect of soil and the vertical component of vibrations. The analysis of RC frames using spectral elements [6] is regarded as an improvement of the preceding model, although the soil-structure interaction is not taken into account. Since the dynamic response of buildings is strongly dependent on dynamic characteristics of soil-structure systems, proper modelling of both the structure and the soil is of crucial significance. One of possible and relatively simple methods is the substructure method [4]. It treats the soil-structure system as a set formed of two substructures, each with entirely different characteristics. One substructure, the building, has finite dimensions, while the other substructure, the soil, is unbounded, i.e. infinite. Buildings are usually modelled using the finite element method, while soil can be modelled by some other numerical techniques, such as the finite element method, the boundary element method, or by using some analytical techniques such as the integral transform method (ITM) [7].

The objective of this paper is to derive a simple numerical model to analyse traffic-induced vibrations of 2D frames, taking into account the soil-structure interaction. The analysis is performed in the frequency domain using the substructure method. The structure is modelled using spectral elements, while the dynamic stiffness matrix of the soil is calculated using the ITM. It is assumed that the foundations are infinitely stiff, and that they rest on the surface of an elastic half-space. The computer program for numerical analysis has been developed in the MATLAB program language [8]. The influence of traffic-induced vibrations on three reinforced concrete frames of different heights (2-storey, 6-storey, and 12-storey) is analysed. Ground vibrations are generated by tram traffic and by a heavy truck crossing rubber obstacles. The effects of frame height and vehicle characteristics on the dynamic response of frames are determined. Finally, principal results are presented.

2. Substructure methods

A soil-structure system subjected to traffic-induced vibrations is presented in Figure 1. The system consists of two substructures, the reinforced concrete frame and the soil. The symbol *s* (*structure*) denotes structural nodes, while *i* (*interaction*) denotes the nodes at the contact between the structure and the soil.

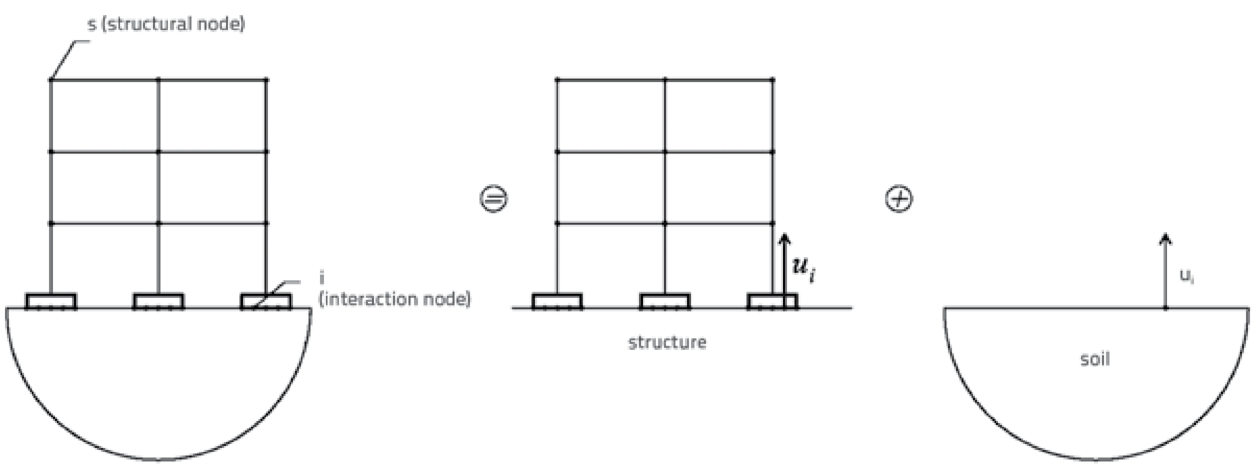


Figure 1. Soil-structure system

In the substructure method, the dynamic equation of the soil-structure system (1) is formulated as a function of total displacement in the frequency domain [5]. It is a system of linear algebraic equations with complex coefficients:

$$\begin{bmatrix} \mathbf{K}_{ss}^{S} & \mathbf{K}_{si}^{S} \\ \mathbf{K}_{is}^{S} & \mathbf{K}_{ii}^{S}+\mathbf{K}_{ii}^{F} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{u}}_{s} \\ \hat{\mathbf{u}}_{i} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{K}_{ii}^{F}\hat{\mathbf{u}}_{i}' \end{bmatrix} \quad (1)$$

where $\mathbf{K}_{ss}^{S}$, $\mathbf{K}_{si}^{S}$, $\mathbf{K}_{is}^{S}$ and $\mathbf{K}_{ii}^{S}$ are the dynamic stiffness sub-matrices of the structure, $\mathbf{K}_{ii}^{F}$ js the dynamic stiffness matrix of the foundations, $\hat{u}_s$ and $\hat{u}_i$ are amplitudes of the displacement vectors of structural nodes (*s*) and interaction nodes (*i*), respectively $\hat{u}_i'$ is the traffic-induced amplitude of the soil displacement vector in the *free field,* i.e. in soil without the structure, Figure 1c [5]. The equation (1) is valid for each frequency ω. All sub-matrices and displacement vector amplitudes are frequency dependent.

3. Spectral element method

In the dynamic FEM analysis, the number of finite elements depends on the highest frequency. Thus, the number of finite elements can be quite considerable for high frequencies. In the one-dimensional spectral element formulation, the dynamic stiffness matrix is obtained using interpolation functions, which are exact solutions of the partially differential equation of motion. Therefore, only one element can exactly represent the dynamic behaviour of a structural member regardless of the vibration frequency. By the use of spectral elements, the number of elements, i.e. the number of unknowns, is reduced considerably, while the accuracy of numerical results is increased.

The dynamic stiffness matrix for the Euler-Bernoulli beam was developed by Kolousek in 1941. Dynamic stiffness matrices for axially loaded beam combined with bending and torsion were developed more recently, [9, 10, 11]. Despite numerous advantages of dynamic analysis based on spectral elements, compared to finite elements, there are only few examples where spectral elements have been used for vibrations analysis of one-element systems or simple frames [6, 12, 13, 14, 15, 16].

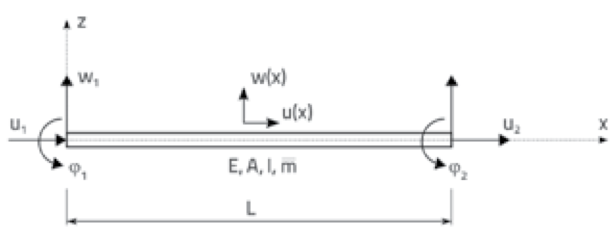


Figure 2. Euler-Bernoulli spectral element

The dynamic stiffness matrix for the Euler-Bernoulli beam, presented in Figure 2, is obtained using dynamic stiffness matrices for the axial and flexural stress of a beam element. The dynamic stiffness matrices for axial and flexural vibrations are obtained from the principle of virtual work:

$$\mathbf{K}_{D} = \int_{V} \mathbf{B}^{T}\mathbf{E}\mathbf{B}dV - \omega^{2}\int_{V} \rho \mathbf{N}^{T}\mathbf{N}dV \quad (2)$$

where **E** is the matrix of elastic constants, **N** is the corresponding matrix of interpolation functions, i.e. $\mathbf{B} = \mathbf{N}'$ for axial deformation, $\mathbf{B} = \mathbf{N}''$ for bending [13]. The interpolation functions satisfy differential equations of displacement:

$$EA\frac{\partial^2 u}{\partial x^2} = \rho A\frac{\partial^2 u}{\partial t^2}, \quad EI\frac{\partial^4 w}{\partial x^4} = -\rho A\frac{\partial^2 w}{\partial t^2} \quad (3)$$

The general solutions of equations are given in the form of Fourier series:

$$u(x,t) = \sum_{n} \hat{u}_n(x,\omega_n)e^{i\omega_n t}, \quad w(x,t) = \sum_{n} \hat{w}_n(x,\omega_n)e^{i\omega_n t} \quad (4)$$

where $\hat{u}_n(x, \omega_n)$ i $\hat{w}_n(x, \omega_n)$ are the spectral components of displacements in *x* and *z* directions, at the frequency ω_n. The interpolation functions **N** for axial and flexural vibrations are obtained by satisfying the governing equation (3) and boundary conditions at the end of elements. The dynamic stiffness matrices for the bar element $\mathbf{K}_{D}^{a}$ and beam element $\mathbf{K}_{D}^{s}$ are obtained from equation (2). Their elements are functions of the element geometry and frequency ω, and are given in the Appendix. A detailed derivation can be found in literature [9, 13, 15].

4. Dynamic stiffness of foundation

The foundation dynamic stiffness matrix has been developed by assuming that the foundation is prismatic and rigid, and that it rests on the surface of an elastic, homogeneous half-space, Figure 3.

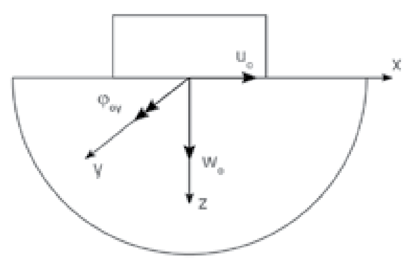


Figure 3. Rigid foundation in half-space

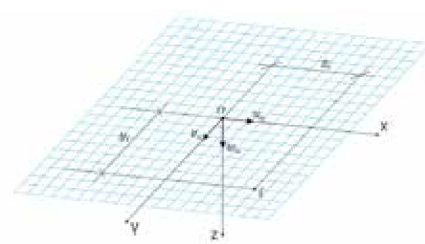


Figure 4. Interaction surface between soil and foundation

Rigid foundation has three degrees of freedom in the *xOz* plane, two displacements and one rotation about centre of the base interface O. Vectors of nodal forces and displacements, $\hat{\mathbf{P}}_o$ and $\hat{\mathbf{u}}_o$ for a frequency ω are

$$\hat{\mathbf{P}}_o = \begin{bmatrix} \hat{P}_x \\ \hat{P}_z \\ \hat{M}_y \end{bmatrix}, \qquad \hat{\mathbf{u}}_o = \begin{bmatrix} \hat{u}_o \\ \hat{w}_o \\ \hat{\varphi}_{yo} \end{bmatrix} \tag{5}$$

The relationship between nodal forces $\hat{\mathbf{P}}_o$ and displacements $\hat{\mathbf{u}}_o$ can be written as follows

$$\hat{\mathbf{P}}_o = \mathbf{K}_o \hat{\mathbf{u}}_o \tag{6}$$

where $\mathbf{K}_o$ is the dynamic stiffness matrix for a rigid foundation

$$\mathbf{K}_o = \begin{bmatrix} k_{xx} & 0 & k_{x\varphi} \\ 0 & k_{zz} & 0 \\ k_{\varphi x} & 0 & k_{\varphi\varphi} \end{bmatrix} \tag{7}$$

The elements of stiffness matrix depend on the frequency ω. For a rigid surface the foundation coupling term is equal to zero, $k_{x\varphi} \cong 0$.
The dynamic stiffness matrix for a rigid foundation is obtained from the dynamic stiffness matrix for a flexible foundation, $\mathbf{K}_{ii}^F$ presented in Figure 4, by equalizing the deformation energy for the flexible and rigid foundations [19]. The dynamic stiffness matrix $\mathbf{K}_{ii}^F$ relates forces $\mathbf{P}_{ii}$ and displacements $\mathbf{u}_{ii}$ at the contact surface by the following equation

$$\mathbf{P}_{ii} = \mathbf{K}_{ii}^F \mathbf{u}_{ii} \tag{8}$$

and can be obtained by inverting the flexibility matrix

$$\mathbf{K}_{ii}^F = \left(\mathbf{F}_{ii}^F\right)^{-1} \tag{9}$$

The element f_{ij} of the dynamic flexibility matrix $\mathbf{F}_{ii}^F$ represents the displacement in node i at the interface due to the unit harmonic force at node j. It can be obtained from the Lame's wave equations in a homogeneous elastic half-space [17]

$$(\lambda + \mu) u_{k,ki} + \mu u_{i,kk} = \rho \ddot{u}_i, \quad i = x, y, z \tag{10}$$

where λ and μ are the Lame's constants, ρ is the mass density of the soil and u_i is the displacement component. The equation is solved by the Integral Transform Method (ITM) [18, 19] which is schematically described in Figure 5.

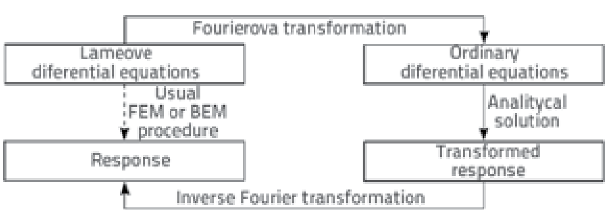


Figure 5. ITM procedure scheme

In ITM, by applying the Helmholtz decomposition [17] and the threefold Fourier transform $x \Leftrightarrow k_x$, $y \Leftrightarrow k_y$, $t \Leftrightarrow \omega$ [20], the partial differential equations are transformed into ordinary differential equations regarding the z direction in the wave number domain:

$$\begin{aligned} &\left(k_p^2 - k_x^2 - k_y^2\right)\hat{\Phi} + \frac{\partial^2 \hat{\Phi}}{\partial z^2} = 0 \\ &\left(k_s^2 - k_x^2 - k_y^2\right)\hat{\Psi}_i + \frac{\partial^2 \hat{\Psi}_i}{\partial z^2} = 0, \quad i = x, y \end{aligned} \tag{11}$$

where Φ=Φ(x, y, z, t) and $\Psi^T = [\Psi_x \, \Psi_y \, \Psi_z]^T$ are scalar and vector displacement potentials, while $k_p = \omega / c_p$ and $k_s = \omega / c_s$ are the wave numbers, and

$$c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \; c_s = \sqrt{\frac{\mu}{\rho}} \tag{12}$$

are velocities of longitudinal (P) and transverse (S) waves, respectively. The potential Ψ_z is assumed to be zero, i.e. Ψ_z=0. Equations are solved analytically in the transformed domain. The result can be returned to the original domain by the inverse Fourier transform. The displacement vector for an arbitrary point at the interface is obtained from equation taking into account the Somerfield radiation condition

$$\begin{Bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{Bmatrix} = \begin{bmatrix} ik_x e^{-\lambda_1 z} & 0 & \lambda_2 e^{-\lambda_2 z} \\ ik_y e^{-\lambda_1 z} & -\lambda_2 e^{-\lambda_2 z} & 0 \\ -\lambda_1 e^{-\lambda_1 z} & -ik_y e^{-\lambda_2 z} & ik_x e^{-\lambda_2 z} \end{bmatrix} \begin{Bmatrix} A_2 \\ B_{2x} \\ B_{2y} \end{Bmatrix} \tag{13}$$

where A_2, B_{2x} and B_{2y} are the integration constants, whereas λ_1 and λ_2 are

$$\lambda_1^2 = k_x^2 + k_y^2 - k_p^2 \;\; \text{i} \;\; \lambda_2^2 = k_x^2 + k_y^2 - k_s^2 \tag{14}$$

Integration constants are determined from the boundary conditions at the interface

$$\begin{Bmatrix} \hat{\sigma}_z \\ \tau_{zx} \\ \tau_{zy} \end{Bmatrix} = \begin{bmatrix} 2k_r^2 - k_s^2 & 2ik_y\lambda_2 & -2ik_x\lambda_2 \\ -2ik_y\lambda_1 & \lambda_2^2 + k_y^2 & -k_x k_y \\ -2ik_x\lambda_1 & k_x k_y & -\left(\lambda_2^2 + k_x^2\right) \end{bmatrix} \begin{Bmatrix} A_2 \\ B_{x2} \\ B_{y2} \end{Bmatrix} \tag{15}$$

where $k_r^2 = k_x^2 + k_y^2$ according to [18].

Elements of the flexibility matrix are obtained numerically [19]. The soil-foundation interface is divided into NxN interaction nodes i, Figure 4. Since the frame is in the plane xOz, the unknown displacements are u and w, in the x and z direction. The flexibility matrix, $\mathbf{F}_{ii}^F$, has $2N$x$2N$ rows and columns. Theoretically, the unit harmonic force has to be applied at each interaction point in x and z direction, and the nodal displacement must be calculated for each frequency ω. These displacements present elements of the flexibility matrix for the proposed frequency. In practice, the displacement field is obtained only once for each frequency ω, due to the unit force at node (i,j), Figure 6a. The displacement field for any other unit force applied in an arbitrary node (*m, n*), Figure 6b, is obtained from the following equation

$$u_z(m+k, n+l) = u_z(i+k, j+l) \tag{16}$$

where *k* and *l* are increments of indices *i* and *j*.

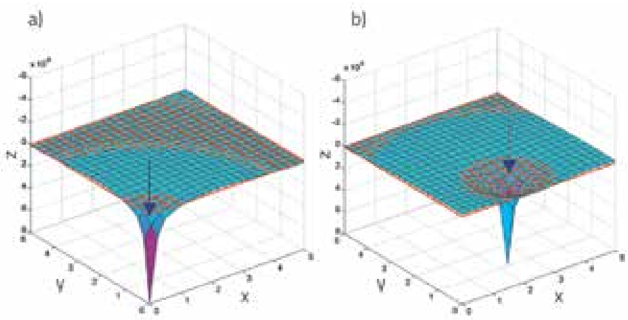


Figure 6. Displacement at the surface of half-space due to unit harmonic force

The dynamic stiffness matrix for the rigid, prismatic, massless foundation, Figure 3, can be obtained from the dynamic stiffness matrix for the contact surface $\mathbf{K}_{ii}^F$ using the following transformation

$$\mathbf{K}_o = \mathbf{a}^T \mathbf{K}_{ii}^F \mathbf{a} \tag{17}$$

where **a** is kinematics matrix

$$\mathbf{a}^T = \left[\mathbf{a}^t{}_1 \cdots \mathbf{a}^t{}_i \cdots \mathbf{a}^t{}_N\right]_{3x2N} \tag{18}$$

The kinematics matrix defines the relation between the displacement vector of all nodes at the interface and the displacement vector of node O. Each sub-matrix $\mathbf{a}_i$ relates the displacement vector at node *i*, $\hat{\mathbf{u}}_i$ točke *i* i and the displacement vector $\hat{\mathbf{u}}_0$ at O

$$\hat{u}_i = a_i \, \hat{u}_0 \tag{19}$$

The sub-matrix $\mathbf{a}_i$ is obtained by kinematics consideration in the following form

$$\mathbf{a}_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -x_i \end{bmatrix} \tag{20}$$

where x_i is the coordinate of node *i*.

The dynamic stiffness matrix $\mathbf{K}_o$ is frequency dependant. Its elements $K_{ij,o}$ are complex numbers. Therefore, each element $K_{ij,o}$ can be written as the sum of the real and imaginary part

$$K_{ij,o}(a_o) = Re\left(K_{ij,o}(a_o)\right) + Im\left(K_{ij,o}(a_o)\right), \; i, j = x, z, \varphi_y \tag{21}$$

where $a_0 = \omega B/c_s$ the dimensionless frequency and *2B* is dimension of the foundation. Dimensionless values of the real and the imaginary part of the dynamic stiffness matrix represent the *impedance functions*. The real part of $Re(K_{ij,o})$ represents the foundation stiffness, while the imaginary part $Im(K_{ij,o})$ represents the radiation damping of the foundation. Impedance functions of a rigid, massless, square foundation in the elastic half-space, are calculated using the presented approach for the frequency range $a_0 \in (0, 2)$, [21]. The impedance functions, presented in Figure 7, are used in the analysis of traffic-induced vibration of frames.

In the frequency domain, the material damping can simply be taken into account using the complex modulus

$$\tilde{E} = E(1+i\eta), \; \tilde{G} = G(1+i\eta) \tag{22}$$

where η is the hysteretic damping coefficient. Eqs. present an additional advantage of the analysis in frequency domain, which enables implementation of a different damping coefficient for particular elements in the numerical model. This is very important for the soil-structure analysis, since the material damping in the soil and in the structure differs considerably. In addition, the radiation damping in the soil is directly taken into account by impedance functions.

6. Analysis of traffic-induced frame vibration

The influence of traffic-induced vibration is analysed on three different two-bay, reinforced concrete (RC) frames of different height: 2-storey, 6-storey and 12-storey, Figure 8. The numerical analysis is carried out using the proposed

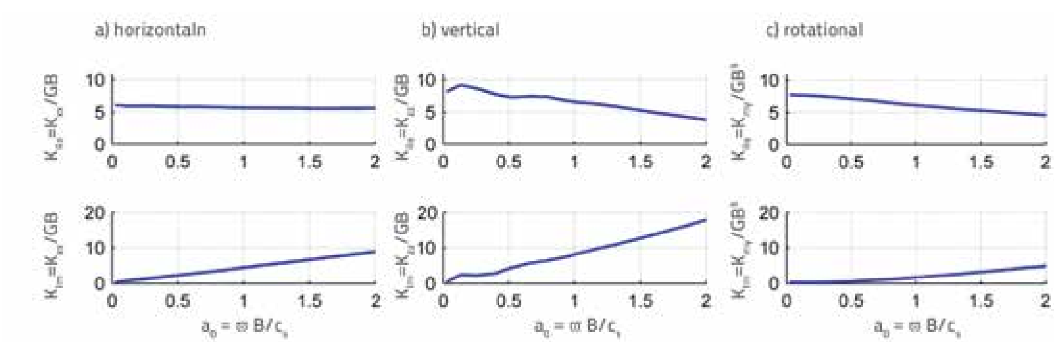
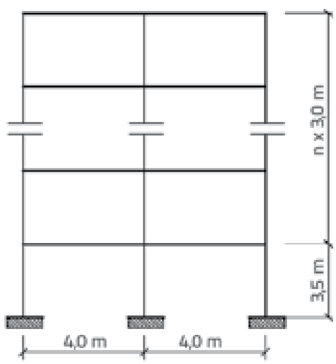


Figure 7. Impedance functions for square foundation in elastic half-space

numerical model and the computer program written in the MATLAB program language [8].



Material properties of frames:

$E = 2\ 10^7$ [kN/m²]
$\rho = 2{,}4$ [kN/m³]
$\nu = 0{,}15$

Heights: 2, 6, 12-storey

Figure 8. Typical frame

The frames bay width is 4 m, the ground floor height is 3.5 m, while the other story height is 3 m. The storey mass equals to 9 t, which is distributed as an additional mass along the beams. The damping coefficient is 5 %. Geometrical properties of the frame from Figure 8 are given in Table 1.

Two different boundary conditions are investigated. In the first case columns are fixed; in the second case columns are founded on the rigid, square, massless foundation measuring 2x2 m.

The properties of the half-space are:
- mass density: $\rho = 2000$ kg/m³
- velocity of S-waves: $c_s = 100$ m/s
- Poisson's coefficient: $\nu = 0.33$
- damping coefficient: 2 %.

A minimum damping coefficient has been adopted so as to emphasize the influence of radiation damping on the soil-structure response.

Table 1. Geometrical properties of frames

Frame	Columns External [cm]	Columns Internal [cm]	Beams [cm]
Two-storey	20x30	25x30	23x40
Six-storey	20x30	25x50 (1-2 floor) 25x40 (3-5 floor) 25x30 (6 floor)	
Twelve-storey	25x40 (1-5 floor) 20x35 (6-8 floor) 20x30 (9-12 floor)	25x80 (1-2 floor) 25x70 (3-4 floor) 25x60 (5-7 floor) 25x50 (8-10 floor) 25x40 (11-12 floor)	

6.1. Influence of soil-structure interaction on natural frequencies

In general terms, soil changes natural frequencies of the system. The literature gives approximate methods for solving natural frequencies of frames flexibly founded on soil. In paper [22] the influence of elastic foundations on high buildings natural frequencies was calculated using the approximate continuum analysis. An advantage of the SEM is that the exact solution of natural frequencies of the frame-soil system can be obtained easily and efficiently, without neglecting the radiation and material damping in the soil. Natural frequencies are obtained from the requirement that the determinant of the soil-structure dynamic stiffness matrix is equal to zero

$$det\mathbf{K} = |\mathbf{K}| = 0 \tag{23}$$

This problem is transcendental. Practically, the number of natural frequencies is infinite, and these values can be found using various searching techniques. In this paper, the natural

Table 2. Natural frequencies for horizontal vibrations [Hz]

Mode	Two-storey frame			Six-storey frame			Twelve-storey frame		
	Fix	Flexible	Δ [%]	Fix	Flexible	Δ [%]	Fix	Flexible	Δ [%]
1	2,37	2,29	3,4	1,08	10,1	6,5	0,62	0,56	9,7
2	7,49	7,4	1,2	3,22	3,11	3,4	1,81	1,74	3,9
3				5,48	5,36	2,2	3,21	3,11	3,1
4				7,95	7,81	1,8	4,7	4,55	3,2
5				10,71	10,55	1,5	6,34	6,14	3,2

Table 3. Natural frequencies for vertical vibrations [Hz]

Mode	Two-storey frame			Six-storey frame			Twelve-storey frame		
	Fix	Flexible	Δ [%]	Fix	Flexible	Δ [%]	Fix	Flexible	Δ [%]
1	15,79	15,44	2,2	12,22	10,55	13,7	8,11	7,9	2,6
2	18,06	21,68	-20,0	13,41	12,8	4,5	13,83	11,58	16,3
3	19,31	50,8	-163,1	16,68	16,24	2,6	16,74	13,57	18,9
4	22,21	57,5	-158,9	18,34	18,09	1,4	18,13	15,93	12,1
5	39,53	63	-59,4	19,25	19,94	1,6	18,83	18,76	0,3

frequencies ω are obtained as the maximum of the following logarithmic function

$$f(\omega) = \log \frac{1}{|\mathbf{K}(\omega)|} \tag{24}$$

Natural frequencies for horizontal and vertical vibrations of rigidly and flexibly founded frames are presented in Tables 2 and 3. The difference Δ between the natural frequencies for fixed base frames and flexibly founded frames is given in percentages.

The soil-structure interaction reduces (SSI) natural frequencies of horizontal vibrations. This reduction is significant in the first mode, and it grows as the number of frame storeys grows, which is due to the effect of base rotation. The biggest difference between natural frequencies for the horizontal vibration of fixed base and flexible foundations is 9.7 % for the first mode of the twelve-storey frame. The influence of soil-structure interaction on natural frequencies of vertical vibration is more pronounced. It depends on the vertical stiffness ratio of frame and soil. Soil reduces natural frequencies for vertical vibrations in the six-storey and twelve-storey frames, but increases natural frequencies in the two-storey frame, except for the first mode. The biggest difference between natural frequencies for vertical vibrations of fixed base frames and flexible founded frames is 163.1 % for the third mode of the two-story frame.

6.2. Traffic-induced frame vibrations

Traffic-induced ground vibrations were measured at the Bulevar kralja Aleksandra, in Belgrade, at the ground level, about 11 m from the tram tracks/road (the average distance between the road and buildings in the street).

Velocity measurements in three orthogonal directions were carried out using the I/O System One and a three-component geophone, Figure 9 [23].

Figure 9. Geophone

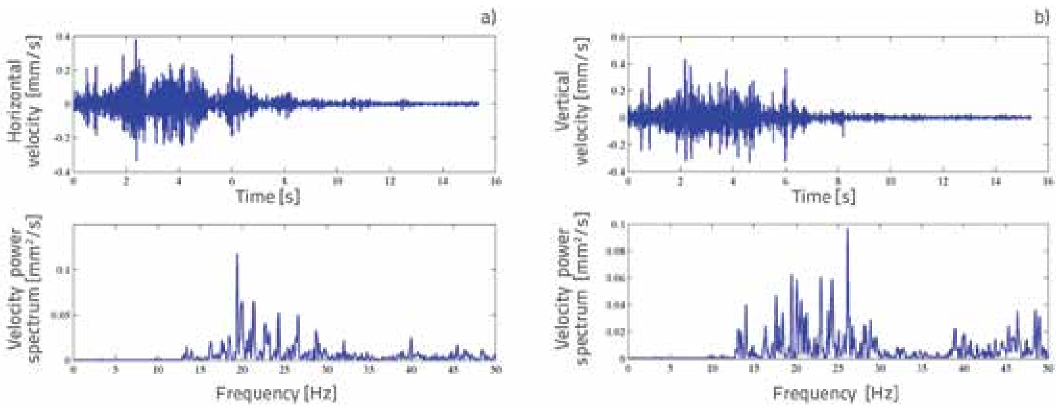


Figure 10. Time history and power spectrum of a) vertical and b) horizontal ground velocity (tram, v=20 km/h)

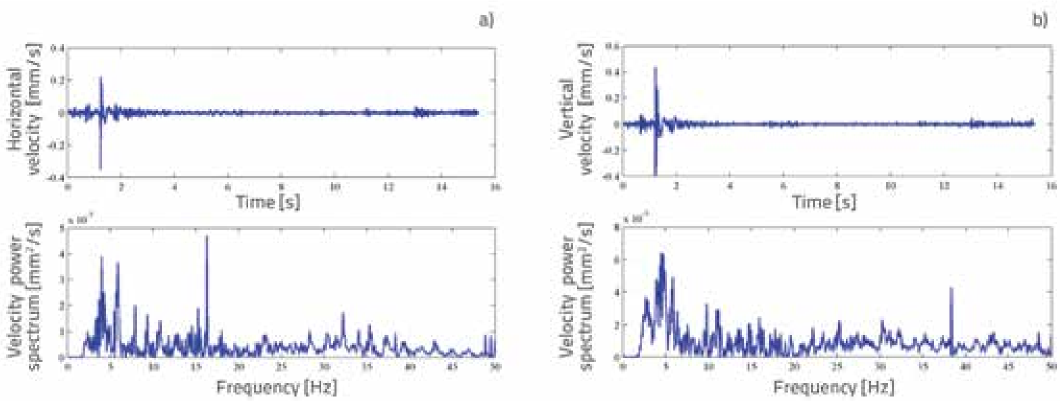


Figure 11. Time history and power spectrum of a) vertical and b) horizontal ground velocity (truck v=50 km/h)

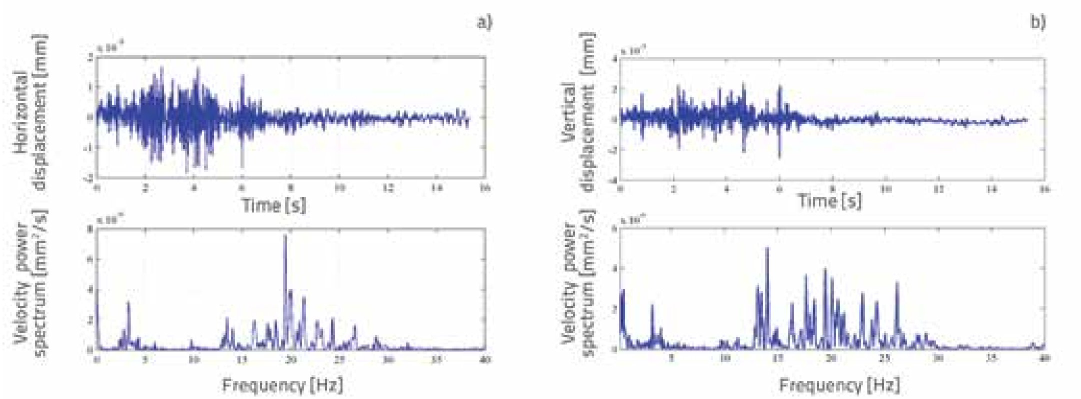


Figure 12. Time history and power spectrum of a) vertical and b) horizontal displacement (tram, v=20 km/h)

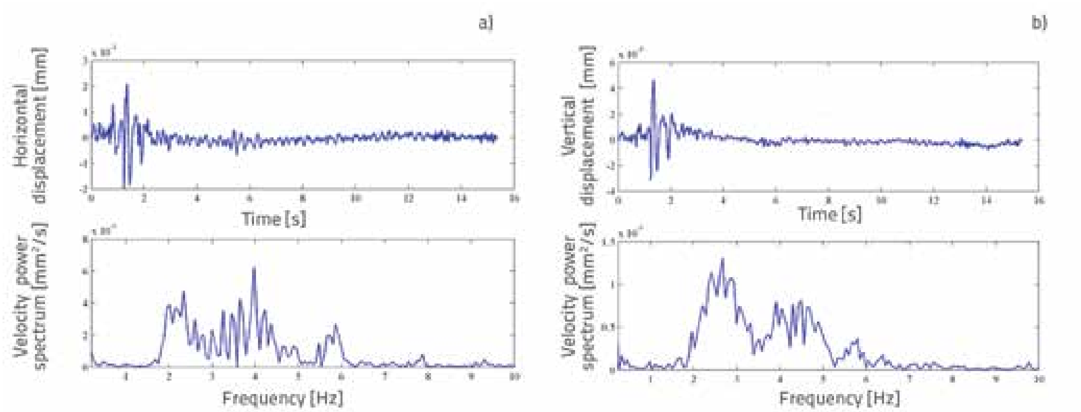


Figure 13. Time history and power spectrum of a) vertical and b) horizontal displacement (truck, v=50 km/h)

The sources of vibrations were:
- normal road traffic,
- 14 t truck, travelling at 50 km/h,
- 14 t truck, travelling at 50 km/h across a 3 cm thick, 3 m long rubber obstacle,
- a tram travelling at 20 km/h.

The highest vibration levels were generated by the tram and the heavy truck running across the rubber obstacle. Therefore, the ground vibrations induced by these two sources were used as an input ground motion in traffic-induced vibration analysis of multi-storey frames.

Velocity time histories and power spectra for horizontal and vertical ground vibrations caused by a tram and a truck are presented in Figures 10–11. Time histories of ground displacements and the corresponding power spectra, obtained by integration of ground velocities, are presented in Figures 12–13. In case of tram traffic, the predominant frequency range for velocity and displacement varies between 17 and 27 Hz for horizontal and vertical vibrations. The predominant frequency range for road traffic induced by a heavy truck crossing the rubber obstacle varies between 2 and 6 Hz for both horizontal and vertical vibrations. Higher amplitudes have been obtained for vertical vibrations.

6.2.1. Numerical results

Due to the specified traffic, the numerical analyses of vibrations of the fixed base frames and flexibly founded frames were carried out using a computer program developed in MATLAB. The frames were subjected to ground displacements due to tram and truck traffic, as shown in Figures 12 and 13. Numerical results are presented as envelopes of horizontal and vertical displacement and velocity of frames in Figures 14–16, where vertical displacements were observed in mid-points of the beams. Analyses were performed with and without soil-structure interaction (SSI).

Horizontal displacements: A truck crossing a rubber obstacle induces larger horizontal displacements than tram traffic for all frames, since the dominant horizontal vibration modes fall into the dominant frequency range for truck traffic (2–6 Hz). The two-storey frame experiences the largest horizontal displacements (fundamental frequency is 2.37 Hz), while the twelve-storey frame has the lowest horizontal displacements.

Vertical displacements: The largest vertical displacements occur on top floors. Vertical dynamic responses of frames are influenced by lower vertical vibration modes. Vertical ground displacements due to the truck crossing the rubber obstacle are larger than vertical ground displacement due to tram traffic in the frame base. However, larger vertical displacements at the top of the fixed bas 2-storey and six-storey frames occur due to tram passage. For flexible founded frames, these displacements are larger due to truck passage.

The soil-structure interaction changes the dynamic response of frames. Generally, horizontal and vertical displacements of the frames are reduced when the soil stiffness is taken into account. The soil influence is larger in case of vertical vibrations, as shown in Figures 14b, 15b, and 16b. If the stiffness ratio between the structure and the soil is larger, the displacements in vertical directions are lower. This is more pronounced if vibrations are induced by tram traffic.

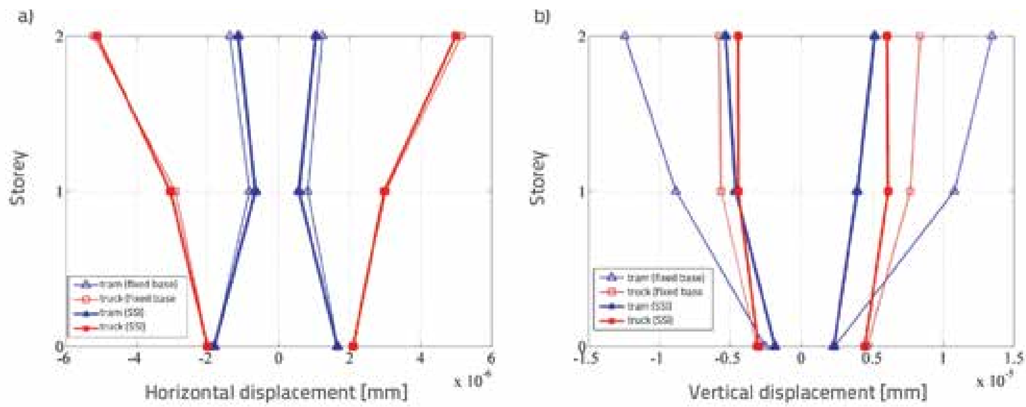


Figure 14. Displacement envelopes of a two-storey frame: a) horizontal displacement; b) vertical displacement

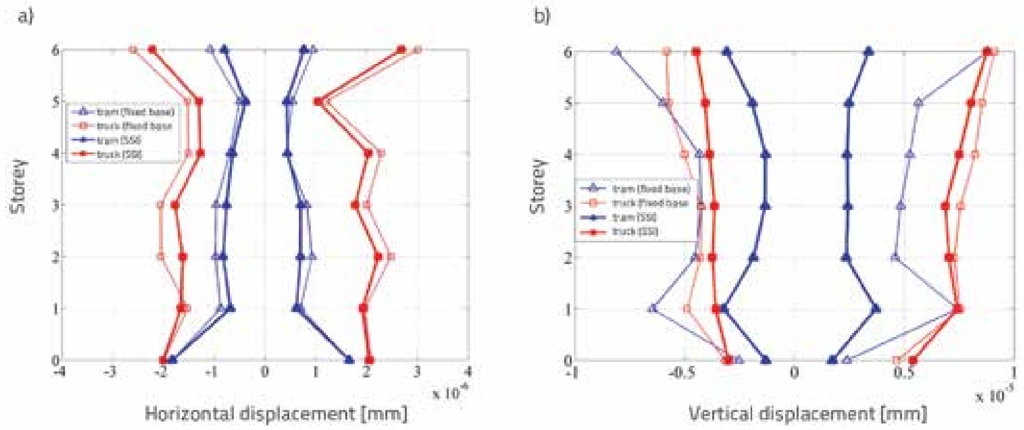


Figure 15. Displacement envelopes of a two-storey frame: a) horizontal displacement; b) vertical displacement

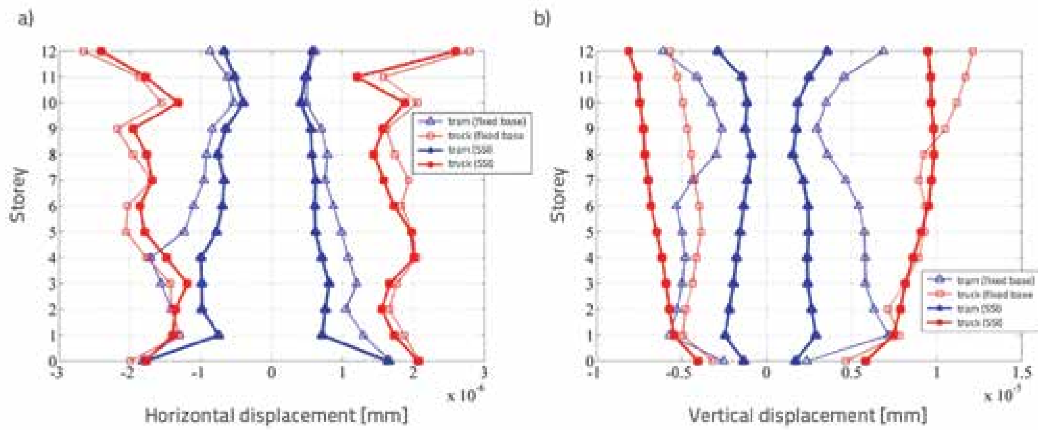


Figure16. Displacement envelopes of a twelve-storey frame: a) horizontal displacement; b) vertical displacement

Humans are sensitive to vibration velocities. Therefore, the velocities are calculated using the well known relationship between the velocity $\dot{u}(\omega)$ and displacement $u(\omega)$ in the frequency domain

$$\dot{u}(\omega) = i\omega \cdot u(\omega) \tag{25}$$

where ω is the angular frequency, and $i = \sqrt{-1}$. The velocity in the time domain $\dot{u}(t)$ is obtained by applying the inverse Fourier transform to the velocity $\dot{u}(\omega)$. Vertical and horizontal velocity envelopes of all frames are presented in Figures 17-19.

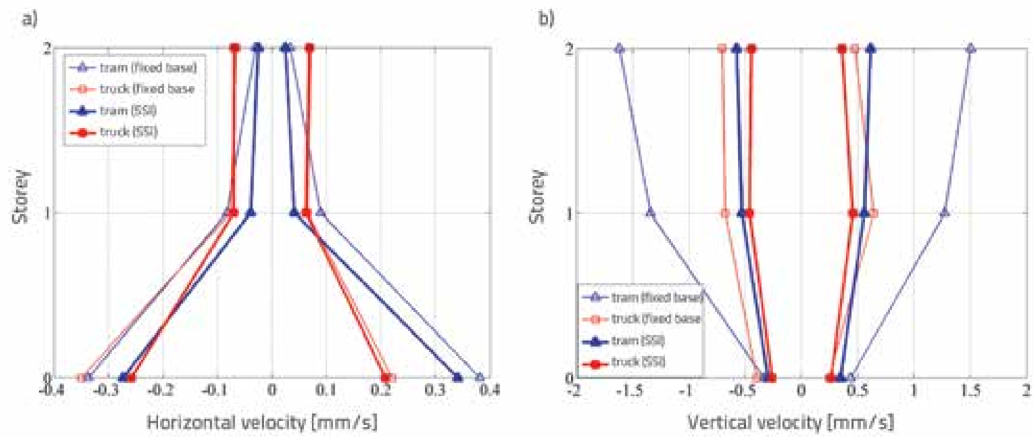


Figure 17. Velocity envelopes of a two-storey frame: a) horizontal velocity; b) vertical velocity

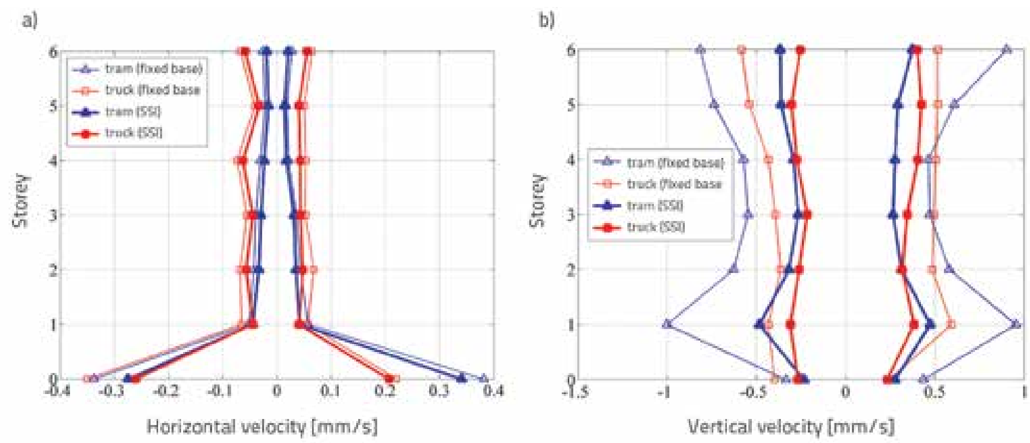


Figure 18. Velocity envelopes of a six-storey frame: a) horizontal velocity; b) vertical velocity

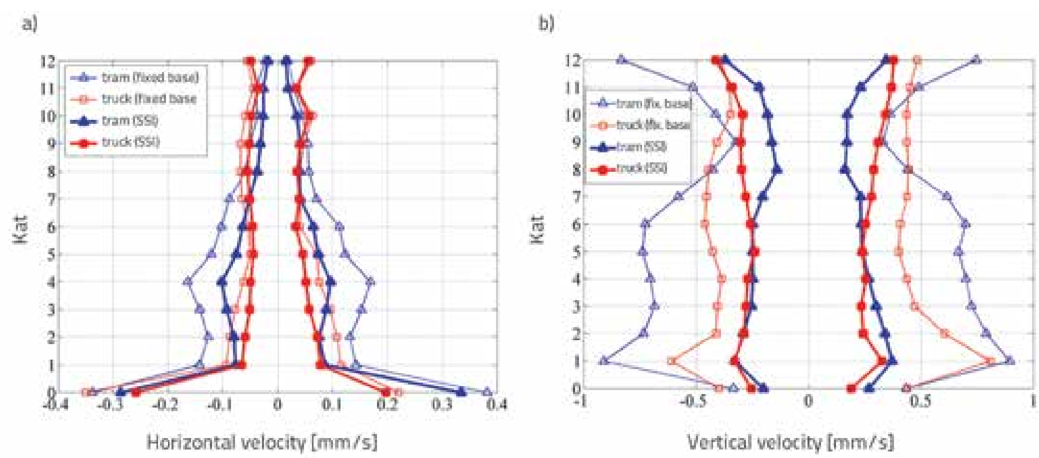


Figure 19. Velocity envelopes of a twelve-storey frame: a) horizontal velocity; b) vertical velocity

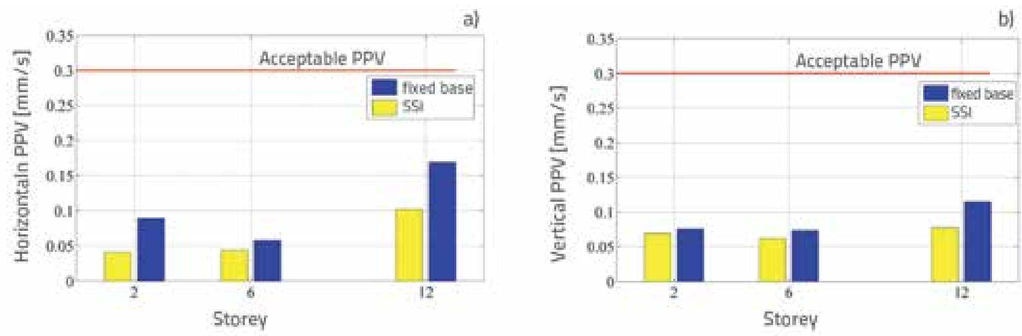


Figure 20. Horizontal PPV of the top floor: a) tram traffic, b) truck traffic, (PPV-peak particle velocity)

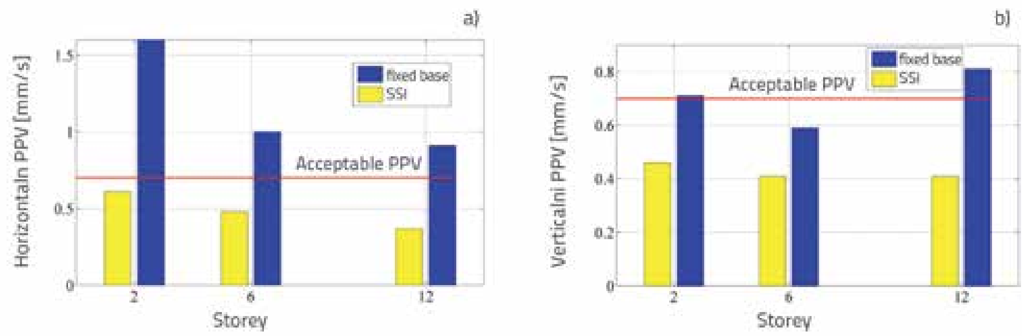


Figure 21. Vertical PPV of the top floor: a) tram traffic, b) truck traffic, (PPV-peak particle velocity)

Horizontal velocities: Tram traffic causes larger ground velocity amplitudes in comparison to truck traffic, but larger velocity amplitudes occur at the top off all frames due to truck passage. Horizontal velocities are attenuated in all frames due to SSI.

Vertical velocities: Frame undergoes larger vertical velocities for tram traffic in comparison to truck traffic, because natural frequencies for vertical vibrations fall into the range of predominant frequencies for vertical tram-induced ground vibrations. Vertical velocities are attenuated in frame structures due to SSI.

Attenuation of displacements and velocities due to SSI are influenced not only by the soil flexibility but also by the damping in soil (radiation and material). We could say that soil behaves as a viscous damper.

7. Assessment of vibration effects on humans

Although traffic-induced vibrations can cause just minor plastic damage to old buildings, they can have annoying effects on humans. Several countries have adopted standards that define the threshold of allowable vibrations in buildings. Since such a standard does not exist in Serbia, the British standard BS: 6472, [24] is used to analyze the effect of frames vibrations on humans. The effect on humans is analyzed using the maximum velocity obtained at the top floor of the frame (PPV-peak particle velocity)

$$PPV = \max\left[\dot{u}(t)\right] \tag{26}$$

Horizontal PPV values do not exceed the perception threshold according to BS: 6472, for both fixed base frames and flexibly founded frames, Fig. 20. Unlike horizontal, vertical PPV values greatly exceed the limit for fixed base frames, but they fall below the limit when the SSI is taken into account, Fig.21. Therefore, the SSI has a positive influence on vibrations, i.e. it reduces vibration velocities and their negative effects on humans in buildings. This effect is more pronounced for "soft" soil (soil with a small *S*-wave velocity).

8. Mitigation measures for traffic-induced vibration

Traffic-induced vibrations are low frequency disturbances. Their frequency content depends on vehicle type and speed, and varies between 1 and 30 Hz. They affect lower vibration modes of buildings, independently of the number of storeys and structural stiffness. Therefore, the measures aimed at reducing traffic-induced vibrations are focused on the reduction of vibrations at the source, and prevention of their transmission from the source to the structure. Preventive measures to reduce traffic-induced vibrations to an acceptable level are:

- maintenance of road surfaces,
- control of traffic flow and speed,
- setting in-ground barriers between the road and the structure,

- increasing the distance between the road and the structure,
- isolation of building foundations, or isolation of floors.

Surface roughness is the main cause of vibrations induced by moving vehicles. The maintenance of road surfaces in urban zones significantly reduces vertical vibration amplitudes. However, this measure calls for substantial financial investments. Therefore the control of traffic flow, i.e. restricting passage of heavy vehicles, and speed limit reductions, are efficient measures that can easily be applied. Placing an open-trench between the pavement and the building is an effective way of stopping wave propagation through the ground. An alternative to open-trenches are trenches filled with a material (concrete) whose stiffness is significantly higher than that of the surrounding soil. Barriers made of sheet-piles measuring 0.5 to 1 m in diameter can also be used. The afore mentioned measures are used for existing buildings.
For new buildings, vibrations can be reduced by increasing the distance between buildings and roads. An expression for surface waves attenuation is:

$$A_2 = A_1\left(\frac{r_1}{r_1}\right)^{1/2} e^{-\alpha(r_2-r_2)} \tag{27}$$

where A_1 and A_2 are the amplitudes of vertical vibration at points 1 and 2, at radii r_1 and r_2 from the source; α is the coefficient that depends on the soil characteristics.
The use of foundation isolation or floor isolation systems to reduce traffic-induced vibration is possible for structures of a very high importance. However, isolation systems are expensive and practically inapplicable for residential buildings.

7. Conclusions

A simple numerical model for the 2D dynamic soil-structure interaction analysis in the frequency domain is presented in this paper. A frame structure is modelled by spectral elements, while the dynamic stiffness of the soil is determined using the integral transform method. The computer program for the analysis of 2D frames was developed in the MATLAB program language. The dynamic response of three different storey frames, influenced to tram traffic and a truck crossing a rubber obstacle, was analysed for the case with SSI and the case without SSI. The major conclusions are:
- SSI reduces natural frequencies of 2D frame vibrations; this reduction is higher for vertical vibrations and low-storey frames;
- Structural response is higher for fixed base frames because soil reduces structural vibrations; if the stiffness of the building is higher compared to soil stiffness, the effect of SSI is more pronounced;
- The SSI effect is more pronounced in vertical vibrations of all frames;
- For fixed base frames, greatest displacements are induced by tram traffic, while for frames with flexible foundations greatest vertical displacements are caused by the truck crossing a rubber obstacle;
- The two-storey frame is most susceptible to vertical vibrations, while the twelve-storey frame is most susceptible to horizontal vibrations, caused by the truck crossing a rubber obstacle;
- PPVs of horizontal vibrations are lower than vibration limits for human perception according to BS: 6472, regardless of the foundation type;
- Vertical PPVs are higher than threshold values for fixed base frames. The SSI reduces the PPVs below this limit, which shows that the SSI has a positive effect on dynamic response.

Due to negative effects of vibrations on humans, the dynamic analysis of traffic-induced ground vibrations should be carried out with great care, taking into account the SSI. Especially sensitive are the cases where a stiff structure is founded on a very soft soil, and when the vehicle velocity and suspension system might cause vibrations close to the predominant vibration frequency of the coupled soil-structure system.
The obtained results show that spectral elements (SE) can be used quite successfully in the analysis of a frame structure-soil system. The proposed model has several advantages when compared to standard numerical models of frame structures. It requires lower number of elements for the analysis of high frequency vibrations in comparison to the finite element method, the influence of the soil can simply be taken into account by adding the dynamic stiffness of soil to the dynamic stiffness of structure and, finally, velocities and accelerations of structural nodes can easily be calculated in the frequency domain from the obtained displacements.
The presented numerical model is simple, efficient, and safe, and can be used in the dynamic analysis of the soil-frame interaction for various dynamic actions: explosions, pile driving, earthquakes, etc. In addition, for certain types of foundations, the dynamic stiffness can be taken directly from literature [25]. In the case of complex foundations, it can be calculated using some of the known methods, like the finite element method, or the boundary element method.
The results of the proposed method should be additionally validated through comparison with results of a detailed building model, or using measurements of an existing building. Based on that, additional conclusions about the efficiency of the proposed model could be reached, which will be the object of further investigations.

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Appendix

Dynamic stiffness matrix for bar

$$\mathbf{K}_\mathbf{D}^a = \frac{EA}{L}\frac{ikL}{1-e^{-i2kL}}\begin{bmatrix} 1+e^{-i2kL} & -2e^{-ikL} \\ -2e^{-ikL} & 1+e^{-i2kL} \end{bmatrix} \quad k = \omega\sqrt{\frac{\rho}{E}}$$

Dynamic stiffness matrix for Euler-Bernoulli beam

$$\mathbf{K}_\mathbf{D}^b = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ & k_{22} & k_{23} & k_{24} \\ sym & & k_{33} & k_{34} \\ & & & k_{44} \end{bmatrix}$$

$$k_{11} = k_{33} = \frac{(1+i)k^3\left(-i-e_2^2+e_1^2\left(1+ie_2^2\right)\right)}{\Delta} \qquad k_{13} = \frac{2k^3\left(ie_2-ie_1^2e_2+e_1\left(e_2^2-1\right)\right)}{\Delta}$$

$$k_{22} = k_{44} = \frac{(1+i)k\ \left(-1+ie_2^2+e_1^2\left(i+e_2^2\right)\right)}{\Delta} \qquad k_{14} = -k_{23} = \frac{2k^2\left(e_2+e_1^2e_2-e_1\left(e_2^2+1\right)\right)}{\Delta}$$

$$k_{12} = -k_{34} = \frac{ik^2\left(e_1^2-1\right)\left(e_2^2-1\right)}{\Delta} \qquad k = \sqrt[4]{\frac{\rho A\omega^2}{EI}}$$

$$\Delta = 1-4e_1e_2+e_1^2\left(1+e_2^2\right), \quad e_1 = e^{ikL}, \quad e_2 = e^{kL}$$

Dynamic stiffness matrix for beam element

$$\mathbf{K}_D = \begin{bmatrix} \mathbf{K}_D^a & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_D^b \end{bmatrix}$$

REFERENCES

[1] Hunaid, O., Tremblay, M.: Traffic-induced building vibrations in Montréal, *Can. J. Civ. Eng.,* 24, pp 736–753, 1997.

[2] Crispino, M., D'apuzzo, M.: Measurement and Prediction of Traffic-Induced Vibrations in a Heritage Building, *Journal of Sound and Vibration,* Vol. 246 (2), pp. 319-335, 2001.

[3] Francois, S., Pyl, L., Masoumi, H.R., Degrande G.: The influence of dynamic soil–structure interaction on traffic induced vibrations in buildings, *Soil Dynamics and Earthquake Engineering,* 27, pp 655–674, 2007.

[4] Hao, H., Ang, T. C., Shen J.: Building Vibration to Traffic-Induced Ground Motion*, Building and Environment,* Vol. 36, pp. 321-336, 2001.

[5] Petronijević, M., Schmid, G.: Metode linearne dinamičke analize sistema tlo-objekat, *Izgradnja*, 62, pp. 541-554, 2008.

[6] Nefovska-Danilović, M., Petronijević, M., Šavija B.: Traffic-induced vibrations of frame structures, *Can. Jour. Civ. Eng.* Vol 40 (2), pp. 157-171, 2013.

[7] Grundmann, H., Trommer, E.: Transform Methods - What Can They Contribute to (Computational) Dynamics, *Computers and Structures*, 79, pp. 2091-2101, 2001.

[8] Matlab, MathWorks Inc. *The Language of Technical Computing*. MATLAB 2011b.

[9] Doyle, J. F.: *Wave Propagation in Structures,* Springer-Verlag, New York, Second edition, 1997.

[10] Banerjee, J. R.: Dynamic stiffness formulation for structural elements: A general approach, *Computers and Structures*, 63, pp. 101-103, 1997.

[11] Lee, U., Kim, J., Leung, A.Y.T.: The Spectral Element Method in Structural Dynamics, *The Shock and Vibration Digest*, 32**,** pp 451-465, 2000.

[12] Petronijević, M.: Određivanje odgovora linijskih sistema primenom brze Fourier-ove transformacije, 18. jugoslovenski kongres teorijske i primenjene mehanike, V. Banja, 30. maj-3. jun, 1988, Srbija C1-29, 1988.

[13] Penava, D.: Vibration analysis of frame structures using the spectral element method, Master thesis, Gradjevinski fakultet, Beograd, 2008.

[14] Penava, D., Sigmund, V., Petronijević, M., Schmid, G.: Vibration analysis of frame structure excited with Rayleigh waves using spectral elements, International Scientific Symposium MODELING OF STRUCTURES, Mostar 13-15.11.2008, BiH, pp. 545-558

[15] Šavija, B.: Dinamicka analiza ramovskih konstrukcija u frekventnom domenu primenom metode spektralnih elemenata, diplomski–master rad, Gradjevinski fakultet Univerziteta u Beogradu, Srbija, 2009.

[16] Šavija, B., Nefovska-Danilović, M., Petronijević, M.: Modalna analiza ramovskih konstrukcija primenom metode spektralnih elemenata, Zbornik radova 13. kongresa DGKS, Zlatibor-Čigota, 22-24.09.2010. Srbija, pp. 365-372

[17] Achenbach, J.D.: *Wave propagation in elastic solid*, North-Holland, Amsterdam-London, 1973.

[18] Rastandi, J. I.: Modelization of Dynamic Soil-Structure Interaction Using Integral Transform-Finite Element Coupling, Lehrstuhl für Baumechanik der Technischen Universität München, 2003.

[19] Radišić, M.: Primijena metoda integralne transformacije (ITM) za odredjivanje pomeranja i napona u tlu usled harmonijskog opterećenja, diplomski–master rad, Gradjevinski fakultet Univerziteta u Beogradu, Srbija, 2010.

[20] Bracewell, R. N.: *The Fourier Transform And Its Applications*, McGraw-Hill, Third Edition, 2000.

[21] Radišić, M., Nefovska-Danilović, M., Petronijević, M.: *Application of integral transform method to calculate impedance functions*, Third Serbian Congress on Theoretical and Applied Mechanics, Vlasina Lake, 5-8 July, 2011, Serbia, pp 965-974, 2011.

[22] Bozdogan, K. B.: Učinak elastičnosti temelja na dinamičko ponašanje zgrade, *GRAĐEVINAR 63* (2011) 4, pp.335-340

[23] Petronijević, M., Nefovska-Danilović, M.: Geodinamička analiza osetljivosti objekata na dejstvo postojećih vibracija prema postojećim standardima i procena njihove osetljivosti na dejstvo lakog metroa, GEOZAVOD i Građevinski fakultet Univerziteta u Beogradu, 2006.

[24] British Standard Institution *Guide to evaluation of human exposure to vibration in buildings* (1-80 Hz): BS6472:1992, London, 1992.

[25] Sieffert, J.G., Ceaver, F.: *Handbook of impedance functions,* Ouest Editions, Presses Academiques, 1992.