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Iterative displacement coefficient method: mathematical formulation and numerical analyses

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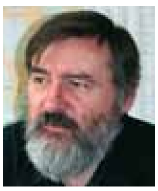

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Preliminary note

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Iterative displacement coefficient method: mathematical formulation and numerical analyses

The mathematical formulation and numerical tests of the originally developed Iterative Displacement Coefficient Method (IDCM) are presented in the paper. The IDCM method is a procedure in the field of seismic analysis of structures that is based on the Nonlinear Static Pushover Analysis. The target displacement level is determined by means of a double iterative procedure developed in the IDCM. The first iterative procedure is carried out simultaneously by forces and displacements, while the second iterative procedure is conducted by correction of the solution obtained for the level of target displacement along a pushover curve. The IDCM method is implemented in the computer code Nonlin Quake TD.

Key words:
target displacement, iterative displacement coefficient method, Nonlin Quake TD

Prethodno priopćenje

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Iterativna metoda koeficijenata pomaka: matematička formulacija i numeričke analize

U radu je prikazana matematička formulacija i numerička analiza originalno razvijene iterativne metode koeficijenata pomaka (IDCM - Iterative Displacement Coefficient Method). IDCM je postupak iz područja istraživanja utjecaja potresa na građevine primjenom nelinearne statičke pushover analize. Razina ciljanog pomaka prema IDCM se određuje primjenom dvostrukog iterativnog postupka. Prvi iterativni postupak provodi se simultano po silama i po pomaku, dok se drugi iterativni postupak provodi korekcijom dobivenog rješenja za razinu ciljanog pomaka po pushover krivulji. IDCM je implementirana u racunalni program Nonlin Quake TD.

Ključne riječi:
ciljani pomak, iterativna metoda koeficijenata pomaka, Nonlin Quake TD

Vorherige Mitteilung

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Iterative Verschiebungskoeffizienten-Methode: mathematische Formulierung und numerische Analysen

In der vorliegenden Arbeit sind die mathematische Formulierung und numerische Analysen der original entwickelten Verschiebungskoeffizienten-Methode (IDCM - Iterative Displacement Coefficient Method) dargestellt. Dieses Verfahren zählt zum Forschungsgebiet der Berechnung von Erdbebeneinwirkungen auf Bauwerke mit Hilfe nichtlinearer statischer Pushover Methoden. Werte der Zielverschiebung werden nach der IDCM durch ein zweifaches Iterationsverfahren ermittelt. Die erste Iteration wird simultan für Kräfte und Verschiebungen durchgeführt, während im zweiten Iterationsverfahren eine Korrektur des ermittelten Ergebnisses für die Zielverschiebung entlang der Kapazitätskurve abgeschlossen wird. Die IDCM ist in das Computerprogramm Nonlin Quake TD implementiert.

Schlüsselwörter:
Zielverschiebung, Iterative Verschiebungskoeffizienten-Methode, Nonlin Quake TD

1. Introduction

A new method based on the nonlinear model of structural behaviour due to seismic action, broadly called the *Nonlinear Static Pushover Analysis* or NSPA, has been developing over the past two decades, and an extensive research aimed at its further improvement is still under way. The NSPA analysis is founded on the modelling of geometrically and materially non-linear behaviour of structures, while treating seismic actions as a static load, explicitly through forces or implicitly through displacements. Therefore, the NSPA idea is based on the principle involving the closest possible approximation in numerical model, and the non-linear material and structural behaviour, while seismic forces in incremental steps are not determined in time domain, but rather in the capacity domain. In such a way, the time needed for numerical simulations is greatly reduced with respect to computing time needed in the *Incremental Nonlinear Dynamic Analysis* or INDA. Also, there is a considerable reduction in the quantity of processing results that are necessary in additional treatment and presentation.

The NSPA analysis is generally conducted in two phases. The first phase is performed using the multi-degree-of-freedom (MDOF) model, while in the second phase the target displacement analysis is done using the single-degree-of-freedom (SDOF) system, or a direct approach is used. From the mathematical aspect, the numerical model used in the NSPA is a discrete model, formulated using the *Finite Element Method* (FEM). It is particularly emphasized that complex numerical models can efficiently be formulated by the finite element discretization, with realistic presentation of important structural features, including relevant details. The NSPA MDOF analysis may be independent from the target displacement analysis in the sense that by using it one could analyse key structural parameters, such as the load capacity, stiffness and ductility in the linear, non-linear or collapse domain. The diagnosis of these parameters is important in evaluation of the current state of structures, and in assessing structural behaviour in future earthquakes. The target displacement analysis may not be independent, since the procedure to determine target displacements depends on the type and character of the pushover curve (PC).

2. Summary of existing methods for target displacement analysis

The development of the NSPA concept and analyses of target displacement of buildings due to seismic actions was initiated more then two decades ago, and official implementations are presented in ATC 40 [1], FEMA 273 [2], Eurocode EC 8 [3], FEMA 356 [4] and FEMA 440 [5] standards. There is nowadays a wide range of NSPA analyses and target displacement analyses but, by their efficiency and superiority, the following ones may be selected:

- *Capacity Spectrum Method* (CSM)
- *Displacement Coefficient Method* (DCM)
- *Equivalent Linearization Method* (ELM)
- *Displacement Modification Method* (DMM)
- N2 *Method*
- *Modal Pushover Analysis* (MPA)
- *Method of Modal Combinations* (MMC)

The Capacity Spectrum Method is implemented in the code ATC 40 [1], while the research related to the development, testing and improvement of this method is presented in [6, 7]. According to the *Capacity Spectrum Method*, the level of target displacements is determined on the basis of intersection of the pushover curve and the demand curve, which is constructed by an iterative procedure. A graphical presentation of the pushover curve, capacity curve, elastic response spectrum, and radial lines representing periods of vibrations, is given in the ADRS format (acceleration-displacement response spectrum). In addition, an important research related to the *Capacity Spectrum Method* with additional improvements, presented as the AutoCSM method, is given in [8, 9]. A further step in the improvement of the capacity spectrum method involves development of the *Adaptive Capacity Spectrum Method* (ACSM), cf. [10], while other significant improvements are given in the Improved *Capacity Spectrum Method* (ICSM), cf. [11-14]. On the other hand, in order to enable a quicker assessment of the target displacement level, the *Non-Iterative Capacity Spectrum Method* (NICSM) was developed, [15]. It is based on the principle of equivalent linearization in the assessment of the required nonlinear structural response. Also, the method of equivalent linearization is presented in FEMA 440, [5], in form of a modified acceleration-displacement response spectrum (MADRS). The displacement coefficient method is based on multiplication of certain coefficients according to FEMA 356, [4], and so the level of target displacement is not determined in an iterative manner. The research related to determination of coefficients applicable in this method is presented in papers [16, 17]. An improvement of this method, represented through the *Displacement Modification Method*, is given in FEMA 440, [5]. The N2 *Method* is implemented in Eurocode EC8, [3], while a more detailed research related to the N2 *Method* is given in [18-20]. The *Modal Pushover Method*, based on reformulation of the *Response Spectrum Method*, turned out to be very superior and efficient in the target displacement analysis, [21]. With respect to this method, the *Multi-Mode Pushover Procedure* (MMPP) was developed, cf. [22, 23]. In addition, the modal combinations method was developed, [24], as well as an alternative with the *Adaptive Modal Combination* (AMC), [25].

3. Iterative Displacement Coefficient Method

The *Iterative Displacement Coefficient Method* (IDCM), originally formulated and presented in this work, is based on the *Displacement Coefficient Method* (DCM), according to FEMA 356, [4]. The DCM method is operated through multiplication of certain coefficients, while in the IDCM method the level of target displacements is determined by the double iterative procedure. The first iterative procedure is simultaneously conducted through forces and displacements, by incremental increase from zero to maximum values. The second iterative procedure is conducted by correcting the solution obtained for target displacement along the pushover curve. In the double iterative algorithm, the first iterative procedure is conducted for all iterations, and then the values are iterated, according to the second iteration procedure, for each iteration from the first iterative procedure. The continuity of the algorithm is achieved by successive implementation of the previously described procedure.

The target displacement level D_t according to the DCM method, FEMA 356, [4], is determined by multiplication of the coefficients:

$$D_t = C_0 C_1 C_2 C_3 S_a \frac{T_{eff}^2}{4\pi^2} g \tag{1}$$

where C_0, C_1, C_2, C_3 are modification coefficients, S_a is the spectral acceleration, and T_{eff} is the effective period of vibrations. C_0 is the modifying coefficient by which the spectral displacement of the equivalent SDOF system is converted into displacement of the top-most node in the MDOF system, which is calculated via the participation factor Γ_1 of the first natural mode and the control node, or according to FEMA 356, [4]. C_1 is the modifying coefficient which represents the ratio of the expected maximum non-linear displacement to the displacement of the linearly elastic response. According to FEMA 356, [4], it is determined as:

$$C_1 = \begin{cases} 1 & \text{for} \quad T_{eff} \geq T_S \\ \dfrac{1 + \dfrac{(R-1)T_S}{T_{eff}}}{R} & \text{for} \quad T_{eff} < T_S \end{cases} \tag{2}$$

where T_S is the characteristic vibration period at the response spectrum changing from the domain of the constant acceleration to the domain of constant velocities, R is the coefficient of the ratio of the elastic bearing capacity and the yield limit bearing capacity. C_2 is the modifying coefficient which represents the effect of the pinched hysteretic shape, stiffness degradation, and strength deterioration at the maximum level of displacement. Values of this coefficient depend on structural performance: immediate occupancy (IO), life safety (LS) or collapse prevention (CP), type of structural system and the value of the initial elastic period of vibrations T_e≤0.1s and T_e≥T_S. C_3 is the modifying coefficient that introduces the increase of displacement due to dynamic *P-Δ* effects. The value C_3=1 is taken as positive stiffness in the non-linear range of behaviour $K_{n,PC}$>0, while it is determined according to FEMA 356 for the negative stiffness in the non-linear range $K_{n,PC}$<0, [4]:

$$C_3 = 1 + \frac{|\alpha_{BC}|(R-1)^{\frac{3}{2}}}{T_{eff}} \tag{3}$$

where α_{BC} is the stiffness ratio in non-linear range $K_{n,BC}$, determined by bilinearization according to the effective stiffness K_{eff}. The effective period of vibrations T_{eff} is determined according to FEMA 356, [4]:

$$T_{eff} = T_e \sqrt{\frac{K_e}{K_{eff}}} \tag{4}$$

where K_e is the initial elastic stiffness. The total base shear force for the level of effective displacement V_{eff} is determined from the intersection of the pushover curve and effective displacement D_{eff}, while the effective stiffness K_{eff} is determined according to:

$$K_{eff} = \frac{V_{eff}}{D_{eff}} \tag{5}$$

In a general case, the domain of possible values of the effective stiffness K_{eff} is given from the initial elastic stiffness K_e to the secant stiffness K_{sec}. The level of the target displacement D_t, as calculated according to previously given expressions, is expected to be in the range of D_{IO}≤D_t≤D_{LS}, and then it is possible to provide a favourable ductile behaviour of the system, while in exceptional cases the level of target displacement may be within the bounds of D_{IO}≤D_t≤D_{CP}. If the previous condition can not be fulfilled, the structure is considered unable to develop the minimum of the necessary ductile behaviour, and so the structural response is unfavourable. The domain of possible values for the $K_{n,BC}$ bilinear curve (BC) is defined by limits in Figure 1, and by the maximum value of the total relative base shear of the structure $(V/W)_{max}$. In certain situations, the maximum value of the total relative base shear $(V/W)_{max}$ needs to be increased by the value of $\Delta(V/W)_{max}$. The alternative is introduced since problems may arise in the quality of bilinearization result in case of NSPA pushover curves with $K_{n,PC}$< 0.

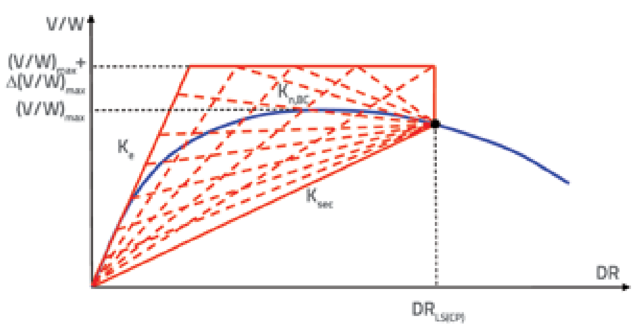


Figure 1. Domain of possible values of $K_{n,BC}$ for $(V/W)_{max}$+$\Delta(V/W)_{max}$

The knowledge of all modifying coefficients is required in the determination of target displacement according to (1), but, as may be seen from previous expressions, the direct determination of C_0 and C_2 is possible, while in some situations C_1 and C_3 could remain unknown, unless it is assumed that C_1 and C_3 are equal to 1. The unknown values in these coefficients are R and α_{BC}. Also, R contains the unknown $V_{y,BC}$ (force at the yield limit for bilinear system), while α depends upon $D_{y,BC}$ (displacement at the yield limit for bilinear system) and $V_{y,BC}$.

In general terms, the process of determination of target displacement of the MDOF system is based on bilinearization by the SDOF system, with additional improvements of non-linear behaviour of SDOF as MDOF system (Figure 2). It is recommended in FEMA 356 [4] to perform balancing of the area below the NSPA pushover curve and the bilinear curve. The solution applied in this method is based on the principle of equal energy of elastoplastic deformations achieved by the NSPA analysis, and presented through the ratio of the bearing capacity-deformation E_{NSPA}, and the energy achieved by elastoplastic deformation from bilinearization E_B. The energy of elastoplastic deformations achieved by the NSPA analyses, E_{NSPA}, is determined from the area whose contour is the polygonal line (pushover curve), vertical line at the position of the target displacement D_t and the abscissa. The term polygon for bilinear system is completely appropriate, since it consists of two lines while, at the first glance, the term seems mathematically incorrectly formulated for the MDOF system. However, since the pushover curve for the MDOF system is generated by connecting discrete values from incremental situations of NSPA analyses, the realistic shape of the pushover curve is also polygonal. The graphical presentation gives the impression of a smooth curve due to a large number of discrete values which are close to one another, and also due to additional interpolations by splines.

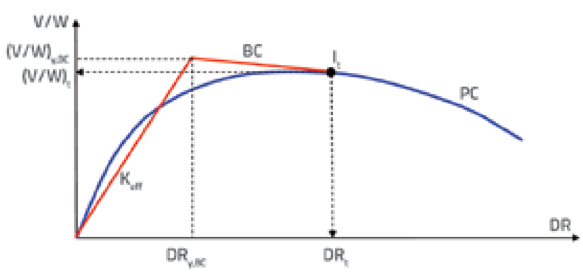


Figure 2. Bilinearization of the NSPA pushover curve

The double iterative algorithm procedure is conducted in several steps. First, the maximum value of the total base shear force is determined from the NSPA analysis:

$$V_{\max} = [V_0, V_n]_{\max},\ (V/W)_{\max} = \frac{V_{\max}}{W} \tag{6}$$

and also the adequate displacement D_{adeq}:

$$D_{adeq} = PC \cap V_{\max},\ DR_{adeq} = \frac{D_{adeq}}{H} \tag{7}$$

where V_{max} is the maximum value of the total base shear force of the structure, W is the total weight, and H is the structural height. In the first phase of the IDCM method, the iterative procedure for $D_{y,BC}$ and $V_{y,BC}$ is conducted in the interval, (Figure 3):

$$D_{y,BC} \in [D_1, D_{\max}],\ V_{y,BC} \in [V_1, V_{\max} + \Delta V_{\max}] \tag{8}$$

where D_{max} is the maximum displacement value at the top of the structure, and ΔV_{max} is the increase of the maximum value of the total base shear force. In this phase of iterations, the coefficients C_1 and C_3 are assumed to be equal to 1, while in the next phase C_1 and C_3 are determined according to expressions (2) and (3), respectively.

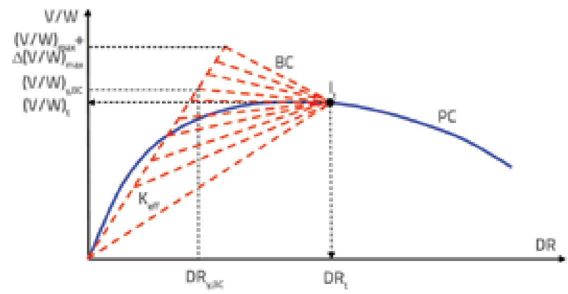


Figure 3. Determination of $D_{y,BC}$ and $V_{y,BC}$ using the first iterative procedure

In the second phase, the iterative procedure for D_t and V_t is conducted using the previously determined $D_{y,BC}$ and $V_{y,BC}$ values (Figure 4). Then the maximum value of the total base shear force is calculated for the bilinear model $V_{max,BC}$:

$$V_{\max,BC} = V_{\max} + \frac{\Delta V_{\max} V_{\max}}{100},\ (V/W)_{\max,BC} = \frac{V_{\max,BC}}{W} \tag{9}$$

and also the adequate displacement $D_{adeq,BC}$:

$$D_{adeq,BC} = \frac{V_{\max,BC}}{K_{eff}},\ DR_{adeq,BC} = \frac{D_{adeq,BC}}{H} \tag{10}$$

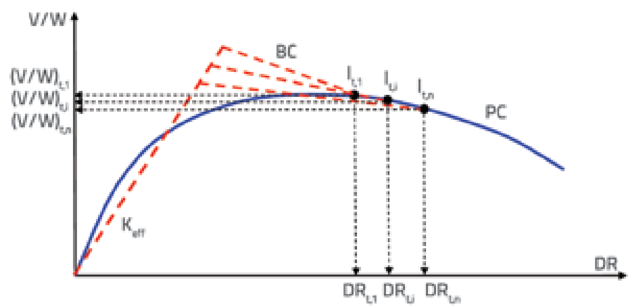


Figure 4. Determination of D_t and V_t applying the second iterative procedure

The increase of the maximum value of the total base shear force is due to the fact that the force at the yield limit for bilinear system $V_{y,BC}$ might be greater than the maximum force obtained by NSPA analysis $V_{y,PC}$. This is almost mandatory for NSPA pushover curves with negative stiffness in the non-linear range $K_{n,PC}$<0. The displacement increment value is determined from the difference of the adequate displacement $D_{adeq,BC}$ for $V_{max,BC}$ and the initial displacement D_0 according to:

$$\Delta D = \frac{D_{adeq,BC} - D_0}{N_{it1}} \tag{11}$$

while the force increment is determined from:

$$\Delta V = \frac{V_{\max,BC} - V_0}{N_{it1}} \tag{12}$$

where ΔD is the increment of displacement, ΔV is the force increment, and N_{it1} is the number of iterations in the first iterative procedure. The initial, the first and iteration number i of displacements at the yield limit of the bilinear system are determined according to:

$$D_{y,BC}^{(1,0)} = D_0,\ D_{y,BC}^{(1,1)} = D_0 + \Delta D,\ D_{y,BC}^{(1,i)} = D_{y,BC}^{(1,i-1)} + \Delta D \tag{13}$$

and the forces according to:

$$V_{y,BC}^{(1,0)} = V_0,\ V_{y,BC}^{(1,1)} = V_0 + \Delta V,\ V_{y,BC}^{(1,i)} = V_{y,BC}^{(1,i-1)} + \Delta V \tag{14}$$

The first index in $D_{y,BC}^{(i,j)}$ and $V_{y,BC}^{(i,j)}$ corresponds to the iteration number in the first iterative procedure, and the second index to the number of iterations in the second iterative procedure. The following iterations for displacement and force at the yield limit are continued by adding the displacement increment and loading increment to previous values until the condition that the last iteration is $i_{\Delta D}=i_{\Delta D}^{(n)}$ and $i_{\Delta V}=i_{\Delta V}^{(n)}$ is fulfilled. In the first iterative procedure, the following values are assumed for the modifying coefficients C_1 and C_3:

$$C_1 = C_1^{(1)} = 1,\ C_3 = C_3^{(1)} = 1 \tag{15}$$

The target displacement for the first iterative procedure is determined as:

$$D_t^{(1)} = C_0 C_2 \frac{T_{eff}^2}{4\pi^2} g \tag{16}$$

and the value of the total base shear force for the first iterative procedure as:

$$V_t^{(1)} = \mathrm{PC} \cap D_t^{(1)} \tag{17}$$

The energy of deformation achieved through bilinear curve $E_B^{(i,i)}$ is determined as follows:

$$E_B^{(i,i)} = 0.5\left(V_{y,BC}^{(i,i)} D_{y,BC}^{(i,i)} + \left(V_{y,BC}^{(i,i)} + V_t^{(i)}\right)\left(D_t^{(i)} - D_{y,BC}^{(i,i)}\right)\right) \tag{18}$$

while the energy of deformation obtained from NSPA analyses $E_{NSPA}^{(i,i)}$ is determined according to:

$$E_{NSPA}^{(i)} = \left[\sum_{i=1}^{t-1} \frac{(V_i + V_{i-1})}{2}(D_i - D_{i-1})\right] + \frac{\left(V_t^{(i)} + V_{t-1}\right)}{2}\left(D_t^{(i)} - D_{t-1}\right) \tag{19}$$

and the difference in energies for iterations from:

$$\Delta E^{(i,i)} = \frac{\left|E_{NSPA}^{(i)} - E_B^{(i,i)}\right|}{E_{NSPA}^{(i)}} (\%) \tag{20}$$

The energy achieved by the NSPA analysis is constant and it is not calculated in iterations conducted in the first iterative procedure. The applicable target displacement for the first iterative procedure is determined by first defining the minimum value of differences in energy from all iterations:

$$\Delta E_{\min}^{(i)} = \left[\Delta E^{(i,1)}, \Delta E^{(i,n)}\right]_{\min} \tag{21}$$

while the other values are taken from the data base of calculated iterations (DBget):

$$D_{y,BC}^{(i)} = \mathrm{DBget}\left(D_{y,BC}, \Delta E, f\left(\Delta E_{\min}^{(i)}\right)\right),\ DR_{y,BC}^{(i)} = \frac{D_{y,BC}^{(i)}}{H} \tag{22}$$

$$V_{y,BC}^{(i)} = \mathrm{DBget}\left(V_{y,BC}, \Delta E, f\left(\Delta E_{\min}^{(i)}\right)\right),\ (V/W)_{y,BC}^{(i)} = \frac{V_{y,BC}^{(i)}}{W} \tag{23}$$

$$E_B^{(i)} = \mathrm{DBget}\left(E_B, \Delta E, f\left(\Delta E_{\min}^{(i)}\right)\right) \tag{24}$$

In the general case, it could be written that $D_{y,BC}$ and $V_{y,BC}$ are in the domain bounded by the intersection of curves:

$$\left(D_{y,BC}, V_{y,BC}\right) = f\left(K_{eff} \cap V_{\max}\right) \cap \left(K_{eff} \cap \mathrm{PC}\right) \tag{25}$$

Once the first iterative procedure is over, the calculation continues with the second iterative procedure in which coefficients C_1 and C_3 are calculated. The coefficient $R^{(i)}$ is determined according to:

$$R^{(i)} = \frac{S_a}{V_{y,BC}^{(i-1)}/W} C_m \tag{26}$$

while the coefficient $C_1^{(i)}$ is determined according to (2), and the coefficient $C_3^{(i)}$ according to (3). When the coefficients are determined, the target displacement level is calculated according to the expression:

$$D_t^{(i)} = C_0^{(i)} C_1^{(i)} C_2^{(i)} C_3^{(i)} S_a \frac{T_{eff}^2}{4\pi^2} g \tag{27}$$

Final values of modifying coefficients and the target displacement level are obtained at the end of all iterations for the first and the second iterative procedure. The first iterative procedure requires more iterations then the second iterative procedure.
The achieved ductility for the level of target displacement μ_t and the maximum available ductility of the system, μ_{max}, is determined through displacements at the yield limit of the system $D_{y,PC}$ of the NSPA analysis:

$$DR_{y,PC} = \frac{D_{y,PC}}{H},\ V_{y,PC} = \mathrm{PC} \cap D_{y,PC},\ (V/W)_{y,PC} = \frac{V_{y,PC}}{W} \tag{28}$$

or using K_{eff}:

$$D_{y,PC} = \mathrm{PC} \cap K_{eff}, V_{y,PC} = K_{eff} D_{y,PC} \text{ for } K_{eff} \neq K_e \tag{29}$$

or by calculating displacements at the yield limit of the system $D_{y,PC}$ according to [26]:

$$D_{y,PC} = DR_y H_{eff},\ DR_y = 0.5\varepsilon_y \frac{L_b}{h_b} \tag{30}$$

where the effective height H_{eff} is calculated according to the expression:

$$H_{eff} = \frac{\sum_{i=1}^{n}(m_i \Delta_i H_i)}{\sum_{i=1}^{n}(m_i \Delta_i)} \tag{31}$$

The design displacement Δ_i of the floor number i is:

$$\Delta_i = \delta_i \left(\frac{\Delta_c}{\delta_c} \right) \tag{32}$$

where:

$$n \le 4; \qquad \delta_i = \frac{H_i}{H}$$
$$n > 4; \qquad \delta_i = \frac{4}{3}\left(\frac{H_i}{H}\right)\left(1 - \frac{H_i}{4H}\right) \tag{33}$$

while for the regular framed systems:

$$H_{eff} = 0{,}7H \tag{34}$$

where ε_y is the steel dilatation at the yield limit, L_b is the beam length (centre to centre of columns), h_b is the beam height, m_i is the mass of i-th floor, and H_i is the height of the floor i. The maximum displacement and the corresponding total base shear force are determined from:

$$D_{\max} = \left[D_0, D_n\right]_{\max},\ DR_{\max} = \frac{D_{\max}}{H},\ V_{adeq} = \mathrm{PC} \cap D_{\max},\ \left(V/W\right)_{adeq} = \frac{V_{adeq}}{W} \tag{35}$$

so, the achieved ductility μ_t for the level of the target displacement and the maximum available ductility of the system μ_{max} are:

$$\mu_t = \frac{D_t}{D_y},\ \mu_{\max} = \frac{D_{\max}}{D_y} \tag{36}$$

Other parameters that are important for the assessment of seismic performances of the system are the secant stiffness $K_{t,sec}$ and the secant period of vibrations $T_{t,sec}$ for the target displacement level:

$$K_{t,\sec} = \frac{V_t}{D_t},\ T_{t,\sec} = T_e \sqrt{\frac{K_e}{K_{t,\sec}}} \tag{37}$$

the optimum period of vibrations according to FEMA 356 [4]:

$$T_{e,opt} = 0.018\left(3.28H\right)^{0.9} \tag{38}$$

the minimum necessary displacement in order to achieve a favourable ductile behaviour of the system:

$$D_{d,\min} = 1.5D_t,\ DR_{d,\min} = \frac{D_{d,\min}}{H} \tag{39}$$

and the (conceptual) damage index for the level of target displacement DI_t [27]:

$$DI_t = \frac{D_t - D_{y,PC}}{D_{\max} - D_{y,PC}} \tag{40}$$

4. Specifics on determination of the target displacement level using the *Nonlin Quake TD*

The *Iterative displacement coefficient method* (IDCM) is implemented in the original software solution *Nonlin Quake TD* (TD – target displacement). Program *Nonlin Quake TD* is a part of the complex software system *Nonlin Quake* for the non-linear performance-based seismic analysis of framed buildings. *Nonlin Quake TD* code is written in the programming language VB/VBA (*Visual Basic/Visual Basic for Application*), [28, 29], and the user-software interaction is done through the corresponding graphical user interfaces (GUI). The initiation of the IDCM method for determining the level of target displacements, using *Nonlin Quake TD*, is performed by correction of discrete values of the pushover curve, obtained by the nonlinear static pushover analysis:

$$D_0 = 0,\ V_0 = 0,\ D_i \to |D_i|,\ V_i \to |V_i|,\ i = 1, \ldots n \tag{41}$$

The conversion into absolute values is performed, since negative values are obtained for V_i in a large number of cases, as the result of numerical solution. The initial elastic structural stiffness K_e is determined according to:

$$K_e = \frac{V_1}{D_1} \tag{42}$$

where V_1 is the total base shear force for the first discrete value, D_1 is the displacement of the top of the structure for the first discrete value. Then the transformation from absolute into relative values is performed:

$$DR_i = \frac{D_i}{H},\ \left(V/W\right)_i = \frac{V_i}{W} \tag{43}$$

The decision about the sign of non-linear stiffness of the MDOF system $K_{n,PC}$ is made by first calculating the tangent stiffness of the system $K_{t,i}$ for two consecutive discrete values of the pushover curve.

$$K_{t,i} = \frac{V_i - V_{i-1}}{D_i - D_{i-1}} \tag{44}$$

with domain selection options:

$$DR_{IO,\min} \le DR_i \le DR_{LS,\max} \text{ i.e. } DR_{IO,\min} \le DR_i \le DR_{CP,\max} \tag{45}$$

as well as:

$$\frac{DR_{IO,\max} + DR_{IO,\min}}{2} \le DR_i \le \frac{DR_{LS,\max} + DR_{LS,\min}}{2} \tag{46}$$

or:

$$\frac{DR_{IO,\max} + DR_{IO,\min}}{2} \le DR_i \le \frac{DR_{CP,\max} + DR_{CP,\min}}{2} \tag{47}$$

Global drifts DR_{IO}, DR_{LS} and DR_{CP} for reinforced framed systems (i.e. structural performance levels), according to SEAOC [30] and FEMA 356 [4], are given by:

$$\begin{array}{llll} DR_{IO,\min} \le 0.5\% & DR_{IO,\max} \le 1\% & D_{IO} = DR_{IO}H & P_{50\%/50} \\ DR_{LS,\min} \le 1\% & DR_{LS,\max} \le 2\% & D_{LS} = DR_{LS}H \text{ for} & P_{10\%/50} \\ \underbrace{DR_{CP,\min} \le 2\%}_{\text{Data for walls}} & \underbrace{DR_{CP,\max} \le 4\%}_{\text{Data for frames}} & D_{CP} = DR_{CP}H & P_{2\%/50} \end{array} \tag{48}$$

while the corresponding total base shear forces for IO, LS and CP performance levels are determined from the intersection of the pushover curve and drifts DR_{IO}, DR_{LS} and DR_{CP}. After selecting the domain and establishing $K_{t,l}$ the weighting coefficients are determined according to:

$$C_{w,i} = \tau \frac{D_i - D_{i-1}}{D_{P,\max} - D_{P,\min}} (\%) \quad \tau = \begin{cases} + & \text{for} \quad K_{t,i} > 0 \\ - & \text{for} \quad K_{t,i} < 0 \end{cases} \tag{49}$$

where $D_{P,max}$ and $D_{P,min}$ are the maximum and the minimum values, respectively, for the selected domain according to (45)-(47). This is followed by summation of all coefficients:

$$C_w = \sum_{i=1}^{n} C_{w,i} \tag{50}$$

and finally, the decision about the sign of $K_{n,PC}$ is made according to the positive or negative value of C_w (a positive stiffness corresponds to positive value of C_w). In certain situations $K_{t,l}$ may significantly change its sign in the non-linear domain, from positive, zero and to the negative value (*saw-tooth force-deflection behaviour)* [31]. On the other hand, the difference in displacements of two consecutive discrete values of the pushover curve may be very significant, and so the general assessment of the sign of the non-linear stiffness may be complicated. This is particularly important at the crossover from the linear into the non-linear domain, and also in the domain of the pre-collapse state. In the research [32-34], which was done using one-dimensional finite elements for modelling framed structures and the *SeismoStruct* software [35], pushover curves obtained by NSPA analyses were presented without a frequent change of stiffness in the non-linear domain. Domains (45)-(47) are selected since it is expected from the structure to develop the non-linear behaviour whose maximum displacement is greater then the LS performance level. If the maximum displacement greater than the LS performance level is not achieved, the lower achieved displacements are considered, and in that case the number of discrete response values is lower. The *Nonlin Quake TD* can iteratively determine the α_{BC} (ratio of the stiffness in the non-linear domain $K_{n,BC}$, determined through bilinearization procedure, to the effective stiffness K_{eff}), while the sign may be different from α_{PC} or through iterations to determine the most favourable case with the same sign as for α_{PC} (Figure 5). The expected (preliminary) global $DR_{t,exp}$ and the interstorey drift $IDR_{t,exp}$ are determined using the modified beta distribution [36]:

$$DR_{t,\exp} = \frac{DR_{\min} + 4DR_t + DR_{\max}}{6}, \quad IDR_{t,\exp} = \frac{IDR_{\min} + 4IDR_t + IDR_{\max}}{6} \tag{51}$$

where DR_{min} and IDR_{min} correspond to the minimum global, i.e. interstorey drift, while DR_{max} and IDR_{max} correspond to the maximum global, i.e. interstorey drift within two consecutive performance levels:

$$DR_{\min} < DR_t \le DR_{\max}, \quad IDR_{\min} < IDR_{t,m} \le IDR_{\max} \tag{52}$$

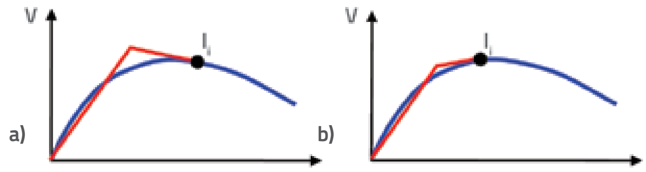


Figure 5. a) α_{BC}<0, α_{PC}<0; b) α_{BC}>0, α_{PC}<0

Minimum and maximum values of the global and interstorey drifts are determined by first determining to which performance level belongs the target displacement. In case of a global drift, the minimum and maximum values are given by (48), while interstorey drifts are presented in [37]:

$$\begin{array}{lll} IDR_{IO,\min} \le 0.2\% & IDR_{IO,\max} \le 0.5\% & P_{50\%/50} \\ IDR_{LS,\min} \le 0.5\% & IDR_{LS,\max} \le 1.5\% \ \text{for} & P_{10\%/50} \\ IDR_{CP,\min} \le 1.5\% & IDR_{CP,\max} \le 3\% & P_{2\%/50} \end{array} \tag{53}$$

All calculated parameters of the IDCM method, and the determined target displacement level, are presented in Figure 6.

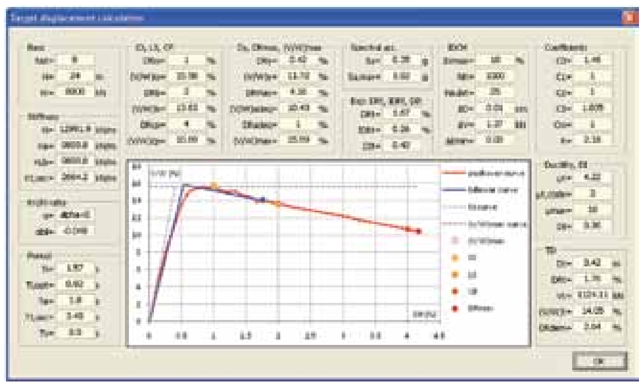


Figure 6. Overview of parameters calculated to determine the target displacement level according to IDCM method

5. Numerical testing

The developed iterative displacement coefficient method, implemented in *Nonlin Quake TD* for NSPA analysis of target displacement, is tested in order to estimate performances, to verify and compare solutions with the solutions obtained by the non-linear dynamic analysis (NDA). The testing was conducted for:

- 15 different NSPA pushover curves generated for standard response models of framed buildings and defining the stiffness, bearing and ductility by accidental choice (using the random function), within empirical boundaries for those parameters, based on the literature and experience, as presented in Figure 7,
- 2D 8-story 4-bay regular framed system according to [34] for seismic action in the plane of the frame,
- SDOF model of the bridge pier 10 m in height, of circular cross-section, 2.5 m in diameter, mass of 1000 t, for

bidirectional seismic action, and designed according to the methodology given in the *Displacement-Based Seismic Design* [26].

First, the values of the coefficient C_3 for 30 NSPA pushover curves with K_n<0 were considered, which were also classified into two groups. The first group includes NSPA pushover curves that were generated for different values of the initial elastic stiffness K_e and ductility levels μ (Figure 7a):

$$\begin{aligned} &DR_y = 0.2 + 0.05i && i = 0,1,\ldots 10 \\ &DR_{max} = 0.5i\left(DR_{IO,max} + DR_{IO,min}\right) && i = 1,1.333 \\ &DR_{LS} = DR_{LS,max} - i && i = 0,1,1.5 \\ &DR_{CP} = DR_{CP,max} - i && i = 0,1,1.5 \end{aligned} \tag{54}$$

$$\begin{aligned} &(V/W)_y = 0.7(V/W)_{max} && - \\ &(V/W)_{max} = 10 + 2.5j && j = 0,1,\ldots 4 \\ &(V/W)_{LS} = 6 + 4j && j = 0,1,\ldots 4 \\ &(V/W)_{CP} = 3 + 4j && j = 0,1,\ldots 4 \end{aligned} \tag{55}$$

While the second group includes the NSPA pushover curves that were generated using the random function while obeying the limits for the linear, non-linear and the collapse domain (Figure 7b):

$$\begin{aligned} &DR_y = 0.1 + 0.6rand() \\ &DR_{max} = 0.7\left(DR_{IO,max} + DR_{IO,min}\right)(0.8 + 0.3rand()) \\ &DR_{LS} = 1.5 + 0.5rand() \\ &DR_{CP} = 2 + 2rand() \end{aligned} \tag{56}$$

$$\begin{aligned} &(V/W)_y = (0.05 + 0.3rand())(V/W)_{max} \\ &(V/W)_{max} = 7.5 + 12.5rand() \\ &(V/W)_{LS} = (V/W)_{max} - 0.3rand()(V/W)_{max} \\ &(V/W)_{CP} = (V/W)_{max} - (0.3 + 0.4rand())(V/W)_{max} \end{aligned} \tag{57}$$

The influence of dynamic *P-Δ* effects is introduced through the coefficient C_3, and since these effects are important for the shape of NSPA pushover curves for K_n<0, the research was performed particularly for this type of curves. Using the IDCM method, the values of the coefficient C_3 were determined for the previously defined NSPA pushover curves in the total of 180 IDCM analyses. After that, series of non-linear regression analyses with different types of functions were performed. An optimum solution was achieved with the exponential function, since the correlation coefficient was the largest (Figure 8). The correlation coefficient r^2 is determined according to [38] as:

$$r^2 = \frac{\sum_{i=1}^{n}\left(C_{3,IDCM,i} - C_{3,IDCM,m}\right)\left(C_{3,Reg,i} - C_{3,Reg,m}\right)}{\sqrt{\sum_{i=1}^{n}\left(C_{3,IDCM,i} - C_{3,IDCM,m}\right)^2 \sum_{i=1}^{n}\left(C_{3,Reg,i} - C_{3,Reg,m}\right)^2}} \tag{58}$$

while the standard deviation is determined as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(C_{3,IDCM,i} - C_{3,IDCM,m}\right)^2}{n-1}} \tag{59}$$

where $C_{3IDCM,i}$ is the discrete value calculated according to the IDCM method, $C_{3IDCM,m}$ is the average value calculated according to the IDCM method, $C_{3Reg,i}$ is the discrete value determined by the regression analysis, while $C_{3RegM,m}$ is an average value determined by the regression analysis. Using the non-linear regression analysis with the exponential function, the expression for the coefficient C_3 was derived in function of the spectral acceleration S_a and for the standardized response model of the NSPA pushover curve, as:

$$C_3 = 1.146e^{0.304S_a} \tag{60}$$

while for the random response function of the NSPA pushover curve it was obtained as:

$$C_3 = 1.304e^{0.282S_a} \tag{61}$$

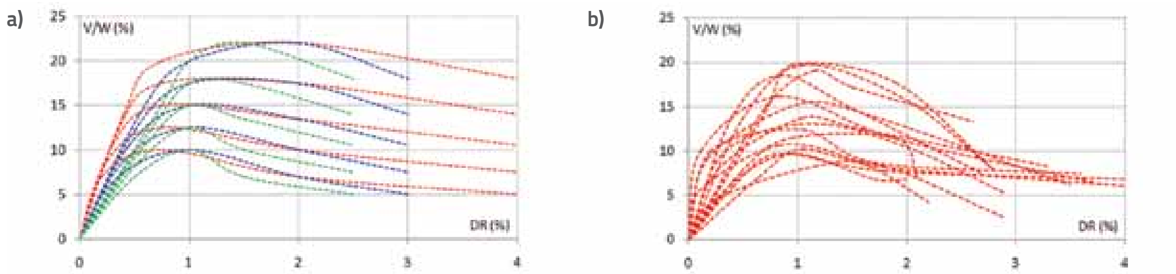


Figure 7. Generated NSPA pushover curves: a) standardized response models; b) random response function, as defined by expressions (54) to (57)

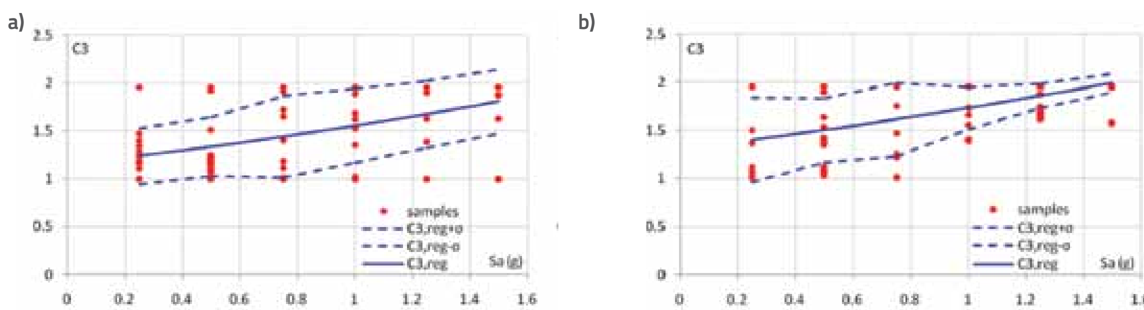


Figure 8. Calculated values of the coefficient C_3 using *Nonlin Quake* TD for NSPA pushover curves: a) standardized response models; b) *random* response function

Finally, the following expression for the coefficient C_3 was derived for all models of the NSPA pushover curves:

$$C_3 = 1.222e^{0.293S_a} \tag{62}$$

With reference to paper [39] where the research was done for the bilinear elastoplastic system, we applied in this research 30 NSPA pushover curves of the real and simulated responses of a building as the MDOF system, and so expressions (60)-(62) may be used directly for practical purposes.
A 2D 8-story 4-bay regular framed system according to [34] (Figure 9) was considered in the second part of the research. To use the IDCM method, a parametric analysis (40 analyses) was performed related to number of iterations N_{it1}, number of iterations $N_{it2,}$ and the additional maximum total base shear force ΔV_{max}:

$$N_{it1} = (50;\ 200;\ 500;2000),\quad N_{it2} = (5;25;50;\ 100),\quad \Delta V_{max} = (0;10)\% \tag{63}$$

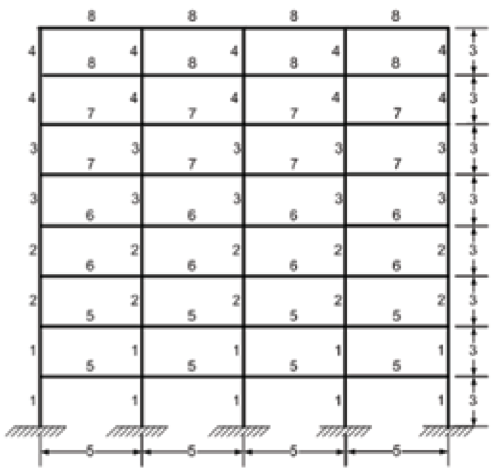


Figure 9. 2D 8-story 4-bay regular framed system [34]

Table 1. Dimensions and reinforcement of the cross sections [34]

Beam No.	Type	Dimensions [cm]	Reinforcement			
			ends		middle	
1	column	40x60	2x5RØ19			
2	column	40x55	2x3RØ19			
3	column	40x50	2x3RØ19			
4	column	40x40	2x3RØ19			
5	beam	30x60	A2	7RØ19	A2	3RØ19
			A1	4RØ19	A1	3RØ19
6	beam	30x55	A2	7RØ19	A2	3RØ19
			A1	4RØ19	A1	3RØ19
7	beam	30x45	A2	7RØ19	A2	3RØ19
			A1	4RØ19	A1	3RØ19
8	beam	30x40	A2	6RØ19	A2	2RØ19
			A1	3RØ19	A1	2RØ19

The preliminary number of iterations in the first iterative procedure N_{it1} can be determined by considering the total base shear force V. If we take that the minimum increment of force increase is m, then the necessary number of iterations N_{it1} is equal to V/m. On the other hand, the number of iterations N_{it1} in the first iterative procedure can also be determined by considering the maximum structural displacement value D_{max}. If one assumes that the minimum increment of displacement increase is n, then the necessary number of iterations N_{it1} is equal to D_{max}/n. It was established through extensive research that the criterion to determine N_{it1} from the ratio V/m is more applicable.
The analogy used in determining the preliminary number of iterations N_{it1} in the first iterative procedure can not be applied for the second iterative procedure, N_{it2}. In this case the parametric analysis and the empirical approach are more reliable. The total of 32 IDCM analyses of target displacements were conducted. Previously, using NDA analysis, the level of target displacement of the frame $DR_{t,NDA}$=1.39% and $(V/W)_{t,NDA}$≈20% was determined for the *Loma Prieta* earthquake (station *Agnews State Hospital*, LP89, PGA_o=0.17g) according to PEER GMDB [40] (Figure 10).

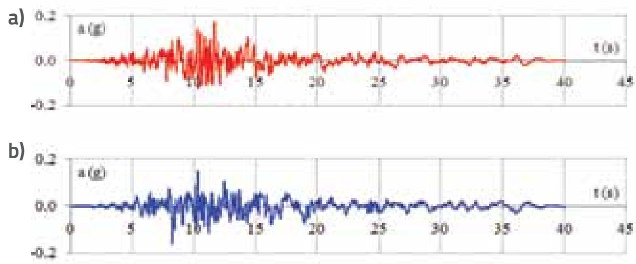


Figure 10. Original non-scaled accelerogram, *Loma Prieta* LP89: a) 1st component; b) 2nd component

Figure 11.a presents calculated global drift values for the target displacement level DR_t, while Figure 11.b) presents calculated relative values of the total base shear force $(V/W)_t$ for the target displacement level, according to IDCM method.

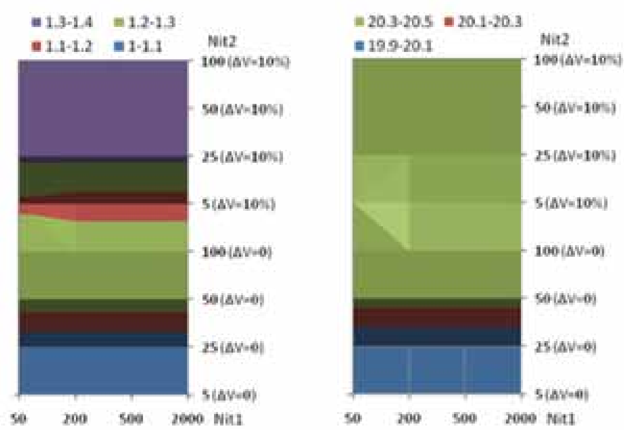


Figure 11. Parametric analysis for variable N_{it1}, N_{it2}, ΔV_{max}: a) DR_t; b) $(V/W)_t$

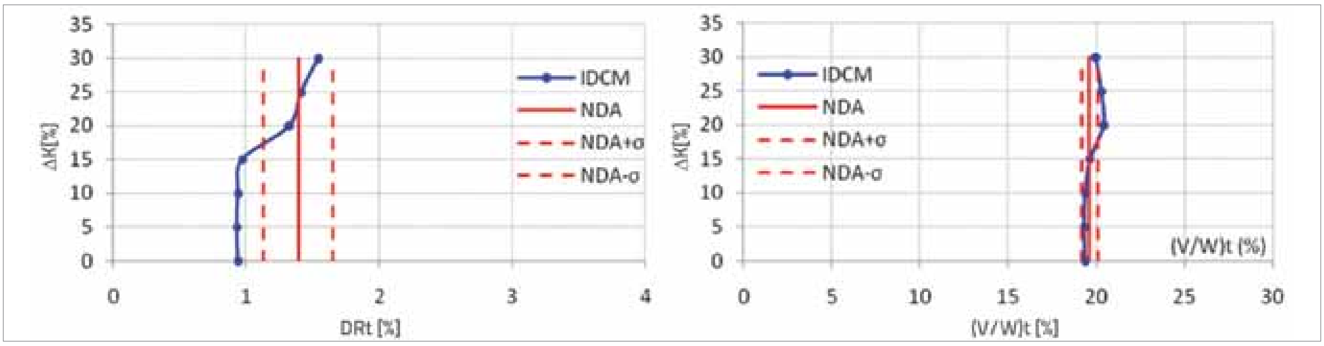


Figure 12. Parametric analysis for variable *ΔK*

The domain of the optimum number of iterations N_{it1} and N_{it2} was determined by comparing global drifts for the level of target displacements $DR_{t,IDCM}$ and $DR_{t,NDA}$ and the corresponding relative value of the total base shear forces $(V/W)_{t,IDCM}$ and $(V/W)_{t,NDA}$ of the parametric IDCM method and the NSPA analysis:

$$\begin{matrix} N_{it1} \in (50;2000) \\ N_{it2} \in (25;100) \end{matrix} \Rightarrow 1250 \leq \Sigma N_{it} \leq 200000 \quad (64)$$

Another parameter which significantly influences the level of target displacement is the value of the reduction of initial elastic stiffness ΔK, through which K_{eff} can be determined. The parametric IDCM method was conducted for ΔK values (Figure 12):

$$\Delta K = (0;5;10;15;20;25;30)\% \quad (65)$$

and the target displacement level according to NDA analysis was considered when applicable.

It can generally be stated that some percentage of increase of the maximum total base shear force ΔV_{max} (about 5 to 10 percent) can be recommended for NSPA pushover curves with K_n<0. It is also advisable to introduce the difference between the initial and effective stiffness $K_e \neq K_{eff}$ when determining the target displacement level. Lower levels of target displacement can be expected in bilinearization procedure if it is assumed in the analysis that $K_{eff}=K_e$.
In the third part of the research, the SDOF model of the bridge pier was considered where, for the IDCM method, the parametric analysis with direct scaling in the peak ground acceleration (PGA) was also conducted. As already stated, the bridge pier is 10m high, of the circular cross section with diametar of 2.5m, and the mass of 1000 t. The pier is designed for the bidirectional seismic action according to the methodology given in the *Displacement-Based Seismic Design* [26]. Analyses were performed for the Loma Prieta (previously presented) two-component accelerogram (fault parallel, FP, and fault normal, FN, accelerograms). Then, the inter-components of the accelerogram were generated for the incremental increase of the angle of $\Delta\theta$=30°, and so the total number of inter-components considered was 12. The procedure for generating accelerogram inter-components is conducted by rotating components FP and FN in the reference coordinate system, where in the final stage one obtains:

$$\begin{bmatrix} a_\theta(t) \\ a_{\theta-90}(t) \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} a_{FP}(t) \\ a_{FN}(t) \end{bmatrix} \quad (66)$$

where $a_\theta(t)$ is the accelerogram for the angle of rotation θ, $a_{FP}(t)$ is the accelerogram for direction FP, and $a_{FN}(t)$ accelerogram for FN direction. The angle ϕ is determined according to:

$$\varphi = \alpha_f - 90° + i\Delta\theta \text{ for } i = 0,\dots 11 \quad (67)$$

where α_f is the angle of fault direction. By analogy to the number of inter-components of the accelerogram, the same number of pushover curves from NSPA analyses was generated and the target displacements were determined using the IDCM method. Fragments of NSPA and NDA pushover curves, generated as the levels of target displacements obtained by parametric analyses, are presented in Figures 13÷18. In case of NDA analyses the fragments are the parts of the INDA pushover curve (*Incremental*

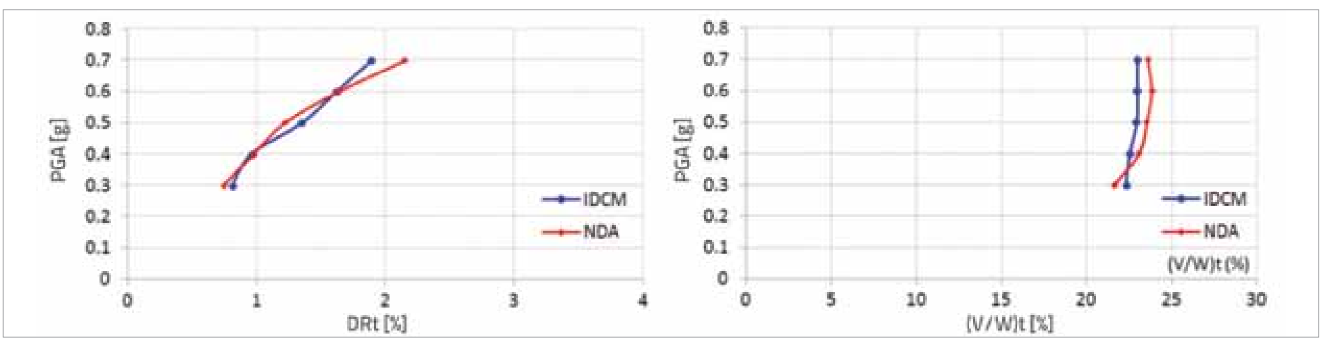


Figure 13. *Pushover* curve fragments for target displacements levels at θ = 0°: a) DR_t; b) $(V/W)_t$

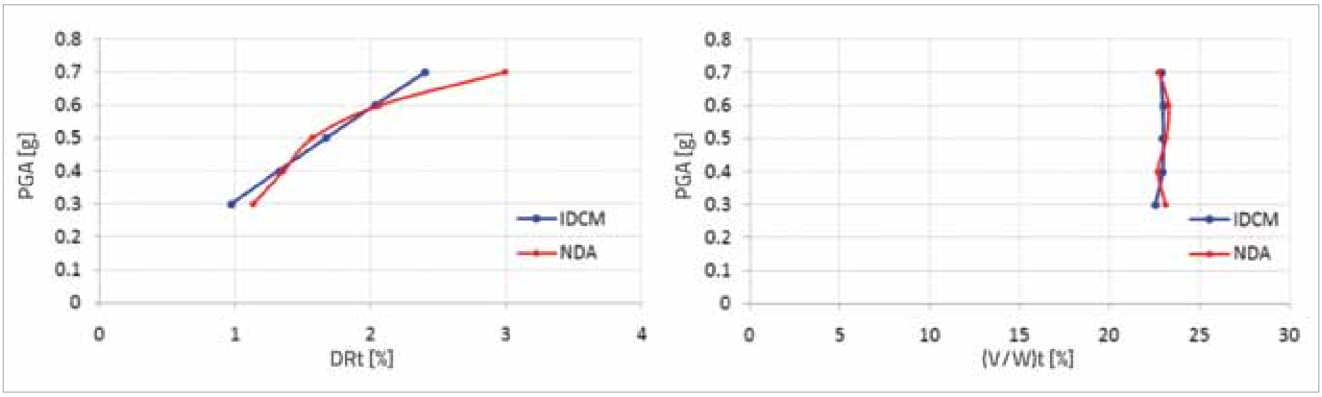


Figure 14. ***Pushover*** **curve fragments for target displacements levels at $\theta = 30°$: a) DR_t; b) $(V/W)_t$**

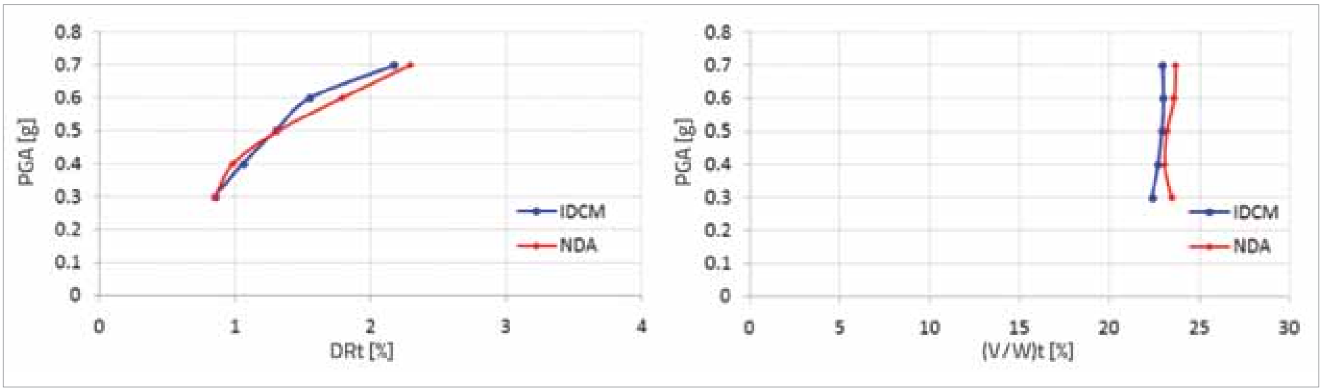


Figure 15. ***Pushover*** **curve fragments for target displacements levels at $\theta = 60°$: a) DR_t; b) $(V/W)_t$**

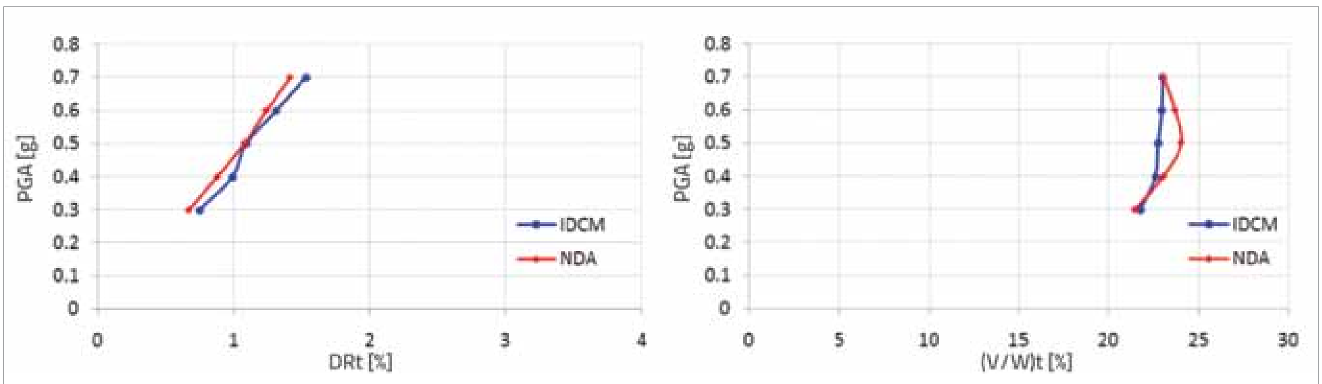


Figure 16. ***Pushover*** **curve fragments for target displacements levels at $\theta = 90°$: a) DR_t; b) $(V/W)_t$**

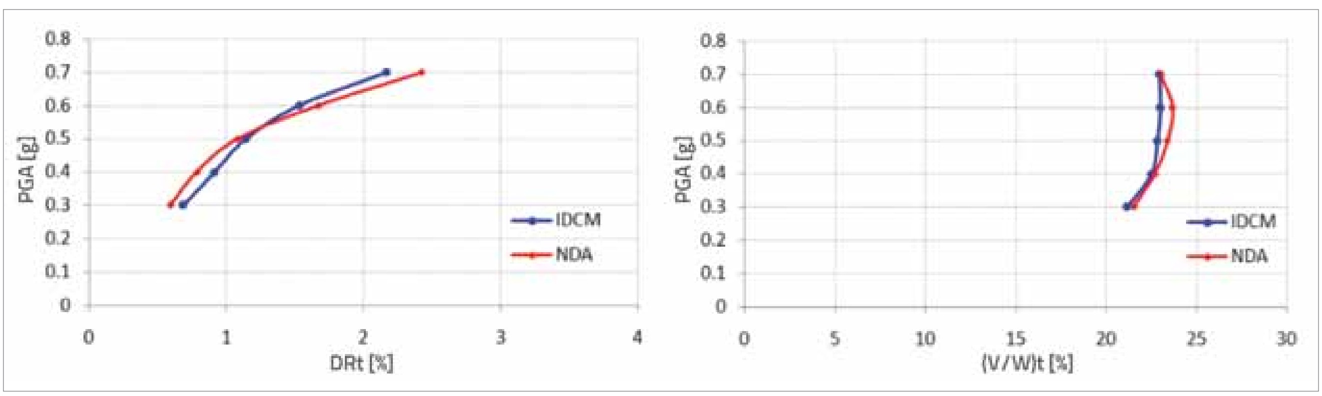
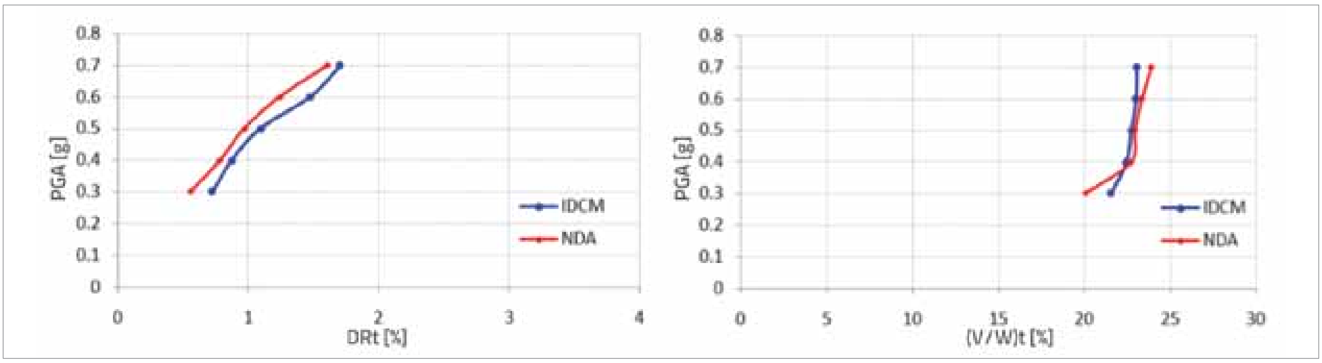


Figure 17. ***Pushover*** **curve fragments for target displacements levels at $\theta = 120°$: a) DR_t; b) $(V/W)_t$**



Slika 18. Isječci *pushover* krivulja za razine ciljanih pomaka pri θ = 150°: a) DR_t; b) $(V/W)_t$

Non-linear Dynamic Analysis), while in case of NSPA analyses the fragments are parts of the NSPA pushover curve for the scaling level of *PGA* = 0.3÷0.7g. The total of 60 IDCM and NDA analyses were conducted. A very satisfactory agreement in drift values was obtained by IDCM and NDA analyses, at almost all target displacement (DR_t) levels. Also, disagreements in the relative total base shear force $(V/W)_t$, obtained by IDCM method, compared to the NDA method, are minimum at all target displacement levels.

6. Conclusion

The mathematical formulation and numerical testing of the originally developed *Iterative Displacement Coefficient Method* (IDCM), with a step by step algorithm, is presented in the paper. The IDCM method is a procedure for estimating earthquake effects on structures using the *Non-Linear Static Pushoevr Analysis*. According to initial idea, the IDCM algorithm is conducted using triple iterations: the first iterative procedure for the effective stiffness, the second iterative procedure generally for forces and displacements, and the third iterative procedure for correcting the solution obtained for the level of target displacements along the pushover curve. By subsequent optimization of the algorithm the number of iterations applied for solution testing was reduced, and so the double iterative algorithm is presented in the final form. The IDCM method is implemented using the originally developed software application *Nonlin Quake TD*, which is a part of the complex software platform *Nonlin Quake* for the non-linear performance-based seismic analysis of framed structures. The IDCM method, as implemented in the *Nonlin Quake TD,* enables a wide spectrum of parameter variations, which directly influences the quality of target displacement obtained. Depending on options implemented in the *Nonlin Quake TD* for the analysis of target displacements, the resulting solutions may vary to a considerable extent. In some situations even numerically unacceptable quasi-solutions may be obtained, and then the robustness of the software points out, in most cases, to the need to correct the number of iterations N_{it1}, N_{it2} and the additional maximum total base shear force ΔV_{max}.

In the first part of the research, the expression for the coefficient C_3, as a function of the spectral acceleration S_a was derived using the non-linear regression with exponential function. 15 NSPA pushover curves generated for the standardized response models of buildings, and 15 NSPA pushover curves generated using the random function, were applied in this research. The coefficient C_3 was determined using the previously presented pushover curves, and also by varying the spectral acceleration in the interval of S_a=0.25÷1.5 g, where the total number of target displacement analyses was 180. In the second part of the research, the parametric analysis was conducted in order to determine the necessary number of iterations N_{it1} and N_{it2} for a realistic 2D 8-story 4-bay regular framed system. The domain of the optimum number of iterations lies within the interval of 1250 to 200000. The supreme value of 200000 iterations may be required in situations when ΔV_{max} = 0 is selected and when, in exceptional cases, the solution can not be obtained with a fewer number of iterations. Sixty IDCM and NDA analyses were conducted In the third part of the research. The comparison of solutions obtained by IDCM and NDA analyses points to a very good agreement in values obtained for drifts DR_t and relative total base shear forces $(V/W)_t$ for the target displacement level.

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