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Design approach for cost-effective hybrid CLT floors

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Research Paper - Subject review

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Design approach for cost-effective hybrid CLT floors

There is a growing research and production interest for the application of lower-cost hybrid cross-laminated timber (CLT) panels composed of timber layers of dissimilar quality. Therefore, an approach for the design of cost-effective hybrid CLT panels in bending, based on the existing analytical and novel finite element procedures, is presented in the paper. The gamma-method, the extended gamma method, and the composite theory are applied in the analysis of square panels, while the finite element model based on the Reddy's full layerwise theory is used for the complex-shape panels in bending. An extensive benchmark technical and economic study is performed for 25 CLT panels, considering various spans, lamination schemes, geometries, and boundary conditions. Cost savings made by using a lower timber class in central zones of CLT panels are highlighted and quantified. This concept shows an economic potential that should be considered during the design and production of CLT panels in structural engineering applications, such as lightweight floor structures.

Key words:
cross-laminated timber, laminate, cost analysis, FEM

Pregledni rad

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Pristup projektiranja ekonomičnih hibridnih CLT podova

Postoji sve veći interes za istraživanjem i izradom hibridnih križno lameliranih drvenih ploča (CLT) koje su sastavljene od slojeva drva različite kvalitete, a koje će smanjiti proizvodne troškove. Iz tog razloga ovaj se rad bavi pristupom projektiranja isplativih hibridnih CLT panela opterećenih na savijanje i to primjenom postojećih analitičkih postupaka te neispitanih postupaka konačnih elemenata. U analizi kvadratnih panela primjenjuju se gama-metoda, proširena verzija gama-metode te kompozitna teorija, a model temeljen na teoriji konačnih elemenata primjenjuje se ako je panel složenog oblika koji je opterećen na savijanje. Provedeno je opsežno istraživanje na 25 CLT panela u kojima se uzimaju u obzir različite dimenzije, sheme laminiranja, geometrije i rubni uvjeti. Ističu se i kvantificiraju uštede troškova koristeći niži razred drva u središnjim zonama CLT panela. Taj je koncept pokazao ekonomski potencijal koji bi se trebao uzeti u obzir tijekom projektiranja i proizvodnje CLT panela u konstrukcijskom inženjerstvu poput laganih podnih konstrukcija.

Ključne riječi:
križno lamelirano drvo, lameliranje, analiza troškova, metoda konačnih elemenata

1. Introduction

Large dimensions of cross-laminated timber (CLT) elements, as well as their easy handling and versatile applicability, have allowed timber industry to expand into markets that had been traditionally reserved for mineral-based solid construction materials. The CLT system was developed in Austria and Germany over the last decades, and it is now rapidly spreading in most European countries. In the early 2000s the construction with CLT increased dramatically, partially driven by green building tendencies, but also due to better efficiency of these elements, code changes, and improved marketing.

The CLT (also known as *X-lam*, *cross lam*, *massive timber* or *Brettsperrholz*) is an innovative material in which timber boards, made of home-grown timber species (mainly spruce), are assembled in layers and glued together crosswise in order to form massive timber wall and floor panels. The cross-lamination method has provided the new material with high stiffness, good mechanical properties, good thermal insulation, and reliable behaviour in the case of earthquake or fire. This makes it applicable for both single unit housing and multi-storey buildings (multi-family apartments, and multi-storey business or administrative buildings). In addition, doors, windows and other openings can be simply positioned and executed using CLT panels, thus allowing for a high level of prefabrication. With regard to building physics, and in comparison with light-weight timber structures, CLT exhibits lower air permeability and a distinctive specific storage capacity for humidity and thermal energy.

CLT production capacities have been developing quite rapidly, at 15–20 % per year, primarily in Austria and Germany. In 2014, the worldwide production volume increased to 625,000 m^3 [1]. There is an ongoing trend of CLT continuously shifting the limits for tall timber buildings. Some of the examples are the 14-storey building *The Tree* in Bergen, Norway [2, 3], *Ho-Ho* in Vienna, or *Mjostarnet* in Norway (an 85.4 m high CLT tower). Besides high-rise construction, CLT also offers great potential with regard to its use in bridge engineering, where it can be used either independently or in combination with other wood and/or steel-based materials in the construction of ribbed and/or box girders [4].

Design procedures for CLT have been regulated via the international European Technical Approvals (ETAs) starting from 2006. The first activities standardizing CLT in Europe began in 2008 and the first European product standard for CLT, EN 16351 [5] has recently passed the formal vote. CLT is to be included in the European timber design code Eurocode 5 [6]. The reason for the slow progress in the development of timber design codes and, in particular, for the difficulties in gaining full understanding of the mechanics of timber materials, lies mainly in the highly complex nature of the wood microstructure [7].

The aim of this paper is to present a design approach for further optimization of the CLT panel behaviour under bending load. The motivation originates from some previous benchmark optimization studies [8, 9] which revealed that 52-77 % of the manufacturing cost of CLT panels originates from the raw material, wood. There is also a research and production interest in the application of hybrid CLT panels in bending, resulting from the previous optimization studies [10-13]. As the lamination concept of CLT enables the use of a lower quality timber in the central area of the cross-section (where middle layers are not affected by maximum stress-strain values), a lower-price timber can be used for this purpose and, hence, the price of hybrid panels can be reduced below that of standard panels.

2. Analytical design methods for CLT panels

2.1. Mathematical formulas and equations

Several analytical procedures are commonly used for the structural design of CLT panels. One of them is the Gamma method (γ-method), which is implemented in Annex B of Eurocode 5 [6]. Other analytical procedures are: (i) composite theory (or "k-method") proposed by Blaβ [14], and (ii) the Kreuzinger's shear analogy method [15]. The above-mentioned methods use a simplified approach, treating the 2D structure as a (1 m wide) beam, and failing to take full advantage of mechanical properties achieved through cross-lamination. They are applicable in the analysis of CLT panels of regular shape. However, in the case of complex geometry, the use must be made of advanced analytical methods, in which panels are considered as orthogonal slabs.

2.2. Gamma and extended gamma method

The Gamma Method (GM) was originally developed by Möhler for beams (having I or T cross section) connected with uniformly spaced mechanical fasteners, along the length of the beam [16]. This method accounts for the horizontal shear deformation occurring in the layers oriented perpendicular to the span direction, and for the vertical shear deformation in longitudinal layers [17]. Longitudinal layers are taken as beam elements connected with *imaginary fasteners* the stiffness of which is equal to that of the rolling shear of cross layers. According to this method, stiffness properties are defined using the effective moment of inertia $I_{0,ef}$ that depends on the section properties and the connection efficiency factor γ. The gamma method can be used for a maximum of 5 layers, and it is recommended for a span-to-depth ratio greater than 30. If CLT panels have more than 5 layers, the Extended Gamma Method (EGM) is required. The main equations of the γ-method, based on Figure 1, are given in this section as follows. The centre of gravity z_s and the net cross section area $A_{0,net}$ are derived according to:

$$z_s = \frac{\sum_{i=1}^{n} \frac{E_i}{E_c} \cdot b_i \cdot d_i \cdot o_i}{\sum_{i=1}^{n} \frac{E_i}{E_c} \cdot b_i \cdot d_i}, \; A_{0,net} = \sum_{i=1}^{n} \frac{E_i}{E_c} \cdot b \cdot d_i \tag{1}$$

where E_i is the Young's modulus of elasticity for ith layer, E_c is the characteristic Young's modulus (here, adopted as the highest

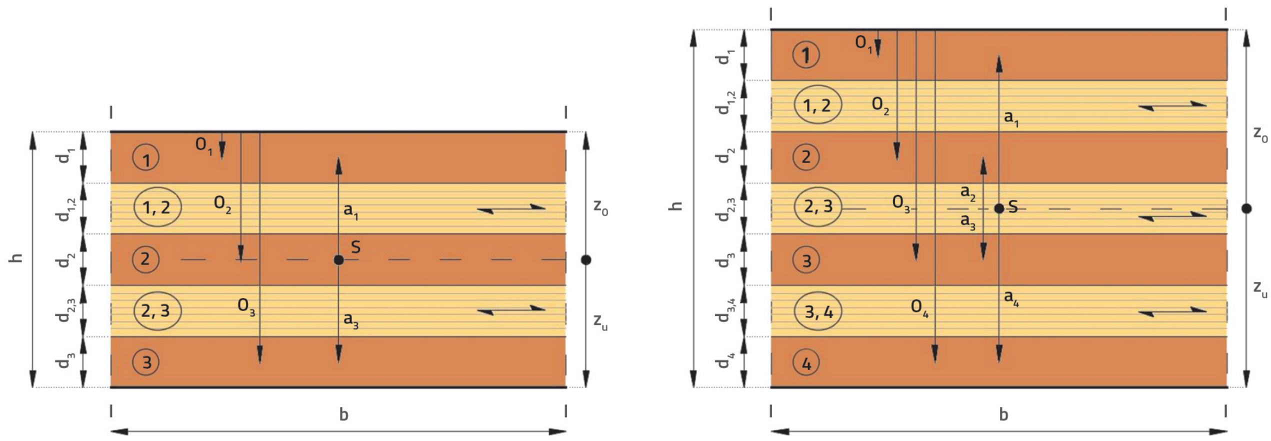

Figure 1. Cross-section and basic input parameters (for γ-method) for a CLT panel with 5 layers (left) and 7 layers (right)

longitudinal Young's modulus of the CLT panel), b_i is the layer width, d_i is the layer thickness, o_i is the distance from the top edge of the CLT panel to the centre of gravity of the i[th] layer. The connection efficiency factors γ for the 5-layer CLT panel are the following (Figure 1, left):

$$\gamma_1 = \frac{1}{\left(1+\frac{\pi^2\cdot E_1\cdot A_1}{l_{ref}^2}\cdot\frac{d_{1,2}}{b\cdot G_{R,12}}\right)},\ \gamma_2 = 1{,}0$$

$$\gamma_3 = \frac{1}{\left(1+\frac{\pi^2\cdot E_3\cdot A_3}{l_{ref}^2}\cdot\frac{d_{2,3}}{b\cdot G_{R,23}}\right)} \quad (2)$$

where E_1 and E_3 are the Young's moduli parallel to the grains of layers 1 and 3, respectively, A_1 and A_3 are the areas of layers 1 and 3, respectively, $d_{1,2}$ and $d_{2,3}$ are the thicknesses of layers 12 and 23, respectively, l_{ref} is the panel span, b is the panel width (1m), and $G_{R,12}$ and $G_{R,13}$ are the rolling shear moduli of layers 12 and 13, respectively. The factor γ can have a value between 0 and 1. The value γ = 1 corresponds to rigid connection, γ = 0 to no connection, while γ = 0.85÷095 [27, 28] are common values for CLT. Distances from the cross-section centre of gravity to the centre of gravity of single layers are:

$$a_1 = \left(\frac{d_1}{2}+d_{1,2}+\frac{d_2}{2}\right)-a_2,\ a_3 = \left(\frac{d_2}{2}+d_{2,3}+\frac{d_3}{2}\right)+a_2 \quad (3a)$$

$$a_2 = \frac{\gamma_1\cdot\frac{E_1}{E_c}b\cdot d_1\left(\frac{d_1}{2}+d_{1,2}+\frac{d_2}{2}\right)-\gamma_3\frac{E_3}{E_c}b\cdot d_3\left(\frac{d_2}{2}+d_{2,3}+\frac{d_3}{2}\right)}{\sum_{i=1}^{n}\gamma_i\cdot\frac{E_i}{E_c}\cdot b\cdot d_i} \quad (3b)$$

Finally, the effective moment of inertia $I_{0,ef}$ can be determined according to the following equation:

$$I_{0,ef} = \sum_{i=1}^{n}\frac{E_i}{E_c}\cdot\frac{b\cdot d^3}{12}+\sum_{i=1}^{n}\gamma_i\cdot\frac{E_i}{E_c}\cdot b\cdot d_i\cdot a_i^2 \quad (4)$$

Using the effective moment of inertia and γ factor, the normal stress ($\sigma_{m,i,d}$) and shear stress ($\tau_{V,i,d}$) can be defined using the following equations:

$$\sigma_{m,i,d} = \pm\frac{E_i}{E_c}\cdot\frac{M_{0,d}}{I_{0,ef}}\left(\gamma_i\cdot a_i\pm\frac{d_i}{2}\right),\ \tau_{V,i,d} = \frac{V_{0,d}\cdot S_{0,i,net}}{I_{0,ef}\cdot b},$$

$$S_{0,i,net} = \sum_{i=1}^{k_L}\frac{E_i}{E_c}b\cdot d_i\cdot a_i \quad (5)$$

where $M_{0,d}$ is the design bending moment, $V_{0,d}$ is the design shear force, $S_{0,i,net}$ is the static moment of area, k_L is an index of the longitudinal layer closest to the position of the centre of gravity as seen from top edge of the cross-section.
According to EC5, the final deflection w_{fin} results from both the instantaneous deflection (w_{inst}) and creep deflection (w_{creep}):

$$w_{inst} = w_G+\sum_{i\geq 1}\psi_{2,i}\cdot w_{Q,i},\quad w_{creep} = k_{def}\cdot w_{inst},$$

$$w_{fin} = w_{inst}\cdot(1+k_{def})\leq\frac{l}{250} \quad (6)$$

where w_G and w_Q are the deflections from permanent and variable loads, respectively, k_{def} is the deformation factor, ψ_2 is the factor for the quasi-permanent value of a variable action and l is the shortest span of the CLT panel. Values k_{def} = 0.8 and ψ_2 = 0.3 are adopted in this paper.
The EGM has to be used for CLT panels composed of more than 5 layers. The method assumes a sinusoidally distributed load and a respective deformation shape of the CLT panel. In the EGM, γ factors are determined using the procedure presented in [23]. After determining γ factors, the procedure for determining the effective moment of inertia, normal stresses, and shear stresses can be carried out using the same procedure as that used in the conventional γ-method.

2.3. Composite theory (k-method)

In this method, the strength and stiffness properties of single layers are taken into account via the composition factors (k_i)

given in [14]. The composition factor is the ratio between the strength or stiffness of the considered cross section and the strength or stiffness of a fictitious homogeneous cross section with the grain direction of all layers parallel to the direction of the stress [14]. The Bernoulli's hypothesis and the linear stress-strain relationship are assumed in the method. This method doesn't take into account shear deformation. It can therefore only be used for span-to-depth ratios greater than 30.

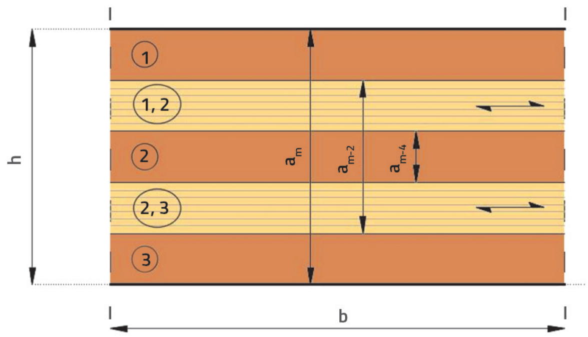

Figure 2. Cross-section of CLT and basic input parameters for k-method

The composition factor k_1 for out-of-plane bending parallel to the grain of outer layers 1 and 3 (Figure 2) can be expressed as follows:

$$k_1 = 1 - \left(1 - \frac{E_{90}}{E_0}\right) \cdot \frac{a_{m-2}^3 - a_{m-4}^3 + \ldots + a_1^3}{a_m^3} \tag{7}$$

In Eq. (7), E_0 and E_{90} are Young's moduli parallel and perpendicular to the grain, respectively. Distances a_i are given in Figure 2. The effective bending stiffness of the CLT panel is defined by the following equation:

$$(EI)_{ef} = E_0 \cdot \frac{b \cdot a_m^3}{12} \cdot k_1 \tag{8}$$

Normal stresses parallel to the grain direction of the outer layers, due to the out-of-plane bending of the CLT panel, at layer interfaces m, m-2 and m-4, respectively, are (see Figure 2):

$$\sigma_m = \frac{M}{(EI)_{ef}} \cdot E_0 \cdot \frac{a_m}{2}, \sigma_{m-2} = \frac{M}{(EI)_{ef}} \cdot E_0 \cdot \frac{a_{m-2}}{2},$$

$$\sigma_{m-4} = \frac{M}{(EI)_{ef}} \cdot E_0 \cdot \frac{a_{m-4}}{2} \tag{9}$$

3. Numerical (finite element) model based on full-layerwise theory

3.1. Model definition

As shown in the previous section, the existing analytical procedures for the design of CLT panels are quite complicated and limited to simple (beam-like or rectangular) geometries, simply supported boundary conditions, and simple loading types. Obviously, there is a necessity to use numerical methods for the analysis of CLT panels in most real cases in engineering practice. The general form of the Reddy's full-layerwise theory (FLWT) [19] was applied for this purpose. The FLWT considers composite laminates (i.e., CLT panels) made of n layers of orthotropic material (k = 1,2,...,n). The total plate thickness is denoted as h (see Figure 3, left). The plate is supported along the portion Γ_u of the boundary Γ and subjected to loads $q_t(x,y)$ and $q_b(x,y)$ acting on either top or the bottom surface of the plate (S_t or S_b). The piece-wise linear variation of all three components of displacement through the plate thickness is imposed, leading to the 3D stress description of all material layers [19].

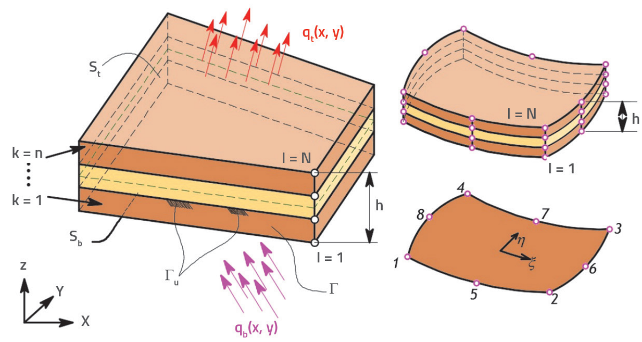

Figure 3. Laminated composite plate with n material layers and N numerical interfaces

The displacement field (u, v, w) of an arbitrary point (x,y,z) of the laminate is given as:

$$u(x,y,z) = \sum_{I=1}^{N} U^I(x,y)\Phi^I(z), v(x,y,z) = \sum_{I=1}^{N} V^I(x,y)\Phi^I(z),$$

$$w(x,y,z) = \sum_{I=1}^{N} W^I(x,y)\Phi^I(z) \tag{10}$$

where $U^I(x,y)$, $V^I(x,y)$ and $W^I(x,y)$ are the displacement components in the I^{th} numerical layer of the plate in the directions x, y, and z, respectively, while N is the number of interfaces between the layers including S_t and S_b. $\Phi^I(z)$ are selected to be linear layerwise continuous functions of the z-coordinate, which can be found in [19]. The linear strain field associated with the displacement field in Eq. (10) can also be found in [19]. The stresses in the k^{th} layer can be computed from the well-known lamina 3-D constitutive equations, which relate the stress and deformation components in the (x,y,z) coordinates. The relation between the (x,y,z) coordinates and the material (1,2,3) coordinates is established through the transformation matrix $\mathbf{T}^{(k)}$ for the k^{th} layer of the laminate [19]. Finally, the Euler-Lagrange governing equations of motion of the FLWT are derived using the principle of virtual displacements. Based on the FLWT, the displacement finite element model (weak form) is derived using an assumed interpolation of the displacement field:

Table 1. Mechanical properties for timber classes C24 and C16

	E_L [N/mm²]	$E_T = E_R$ [N/mm²]	$G_{LT} = G_{LR}$ [N/mm²]	G_{RT} [N/mm²]	ν_{LT}	ν_{LR}	ν_{RT}
C24	11000	370	690	69	0,49	0,39	0,64
C16	8000	270	500	50	0,49	0,39	0,64

$$U^I(x,y)=\sum_{j=1}^{m}U_j^I\psi_j(x,y), V^I(x,y)=\sum_{j=1}^{m}V_j^I\psi_j(x,y),$$
$$W^I(x,y)=\sum_{j=1}^{m}W_j^I\psi_j(x,y) \quad (11)$$

In Eq. (11), m is the number of nodes per 2-D element, U_j^I, V_j^I, W_j^I are the nodal values of displacements U^I, V^I and W^I, respectively, in the j^{th} node of the 2-D element, while $\psi_j(x,y)$ are the 2-D Lagrange interpolation polynomials associated with the j^{th} element node. Obviously, the layered finite elements require only C^0 continuity of generalized displacements along element boundaries, because only translational displacement components are adopted as the nodal degrees of freedom.

The finite element model has been recently implemented within the original finite element code FLWTFEM [20], written by the second author and available online through the GitHub repository [21]. In the paper, quadratic serendipity (Q8) layered quadrilateral elements with reduced integration have been selected from the FLWTFEM library (Figure 3, right), which proved to give reliable results in the bending analysis of CLT [20]. The assumed piecewise linear interpolation of displacement field through the laminate thickness provides discontinuous stresses across the interface between adjacent layers. Once the nodal displacements are obtained, the stresses σ_x, σ_y, σ_z, τ_{yz}, τ_{xz} and τ_{xy} can be computed from the constitutive relations of every layer. The stresses are computed at layer interfaces, and they are discontinuous due to the orthotropic nature of every layer. Since the interlaminar stresses τ_{yz}, τ_{xz} and σ_z calculated in this way do not satisfy continuous distribution through the laminate thickness, they are re-computed by means of the original stress recovery algorithm presented in [22, 23], averaging the interlaminar stresses within each layer, and utilizing 3D equations of equilibrium in terms of stresses.

3.2. Model verification against common design procedures

In this subsection, the model is verified against the design procedures commonly used for CLT (described in Section 2). The goal is to prove the potential of the novel FEM-based procedure for practical use in engineering calculations. A more detailed validation study is presented in [22].

Square CLT panels, simply supported along *B*-sides and free along *L*-sides, are considered. The plates are exposed to the uniformly distributed load Q = 6 kN/m² on the top surface. The first plate (Figure 4a), composed of 5 layers (h_i = 3cm), measures $L = B = 5 \times 5$ m and the overall thickness is h = 15 cm (L/h = 33.3). The second plate (Figure 4b) measures $L = B = 6 \times 6$ m and the overall thickness is h = 21 cm (L/h = 28.5). It is composed of 7 layers (h_i = 3cm). Each layer is considered as a C24 unidirectional lamina, with the material properties given in Table 1, adopted according to [24, 25].

Stress prediction in the first plate is performed using both the γ- and k- methods, while the second plate is calculated according to the EGM due to presence of 7 layers. In all analytical procedures, the considered CLT panels are analysed as equivalent 1m wide beams. In both the γ-method and the EGM, stiffness properties are defined using the effective moment of inertia $I_{0,ef}$. In the *k*-method, the strength and stiffness properties of single layers are taken into account via composition factors k_i.

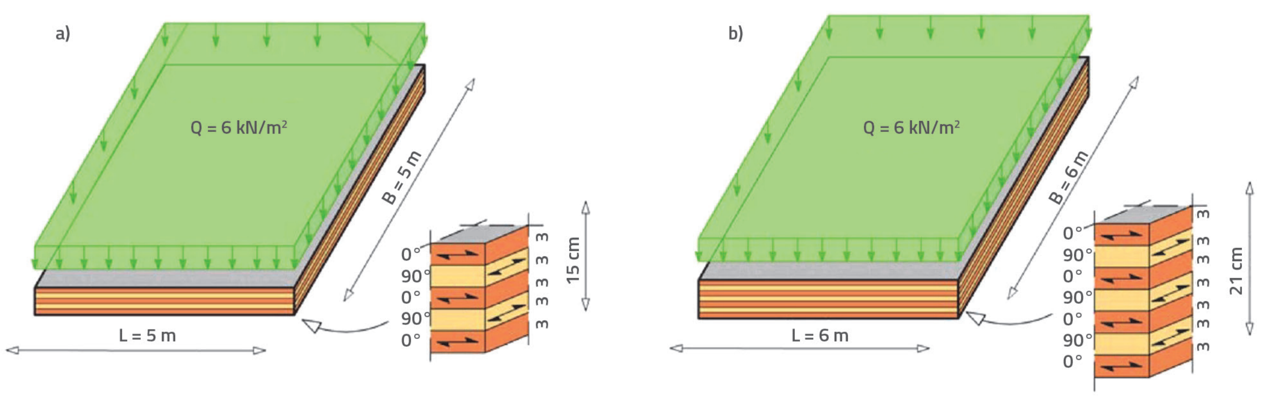

Figure 4. Considered square CLT panels with: a) five; b) seven layers used for model verification against common design procedures

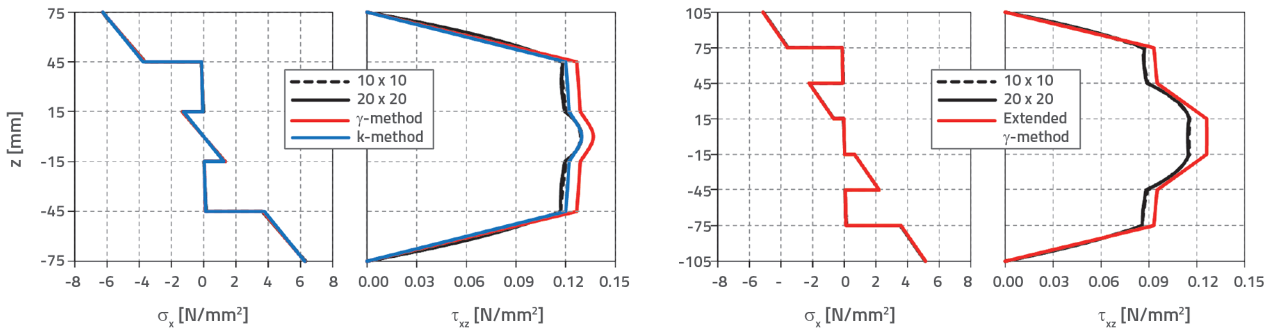

Figure 5. Stresses σ_x (L/2, B/2) and τ_{xz} (0, B/2) in 5-layer CLT panel (left) and 7-layer CLT panel (right) under uniformly distributed load of Q = 6kN/m², considering different design procedures and mesh densities

In the finite element model, boundary conditions are prescribed in edge nodes: $V = W = 0$ for edges parallel to B. Two different mesh sizes are considered in both models: 10 x 10 and 20 x 20 Q8 finite elements with reduced integration.

Distributions of the stress components σ_x and τ_{xz} are shown in Figure 5, along with the results obtained using the γ-method, k-method, and EGM. The distribution is calculated in the points of the panel where stresses reach maximum values.

Obviously, normal stress σ_x exhibits the correct characteristic discontinuous shape, with considerably different slopes in soft and stiff layers. An excellent agreement for σ_x is obtained for both 5- and 7-layer panels when comparing numerical and analytical procedures (Figure 5). The transverse shear stress distributions obtained in both the analytical procedures and the finite element models based on FLWT exhibit the laminate-specific course. A slight overprediction of τ_{xz} is detected in analytical procedures, which is on the safe side when applied in engineering calculations. In the finite element model, it is obvious that even the coarse mesh accurately predicts distribution of τ_{xz}.

It is obvious that the presented finite element model could reliably be used for predicting stress in both thin and thick CLT panels. In the next section, the FEM-based model will be applied for bending analysis of CLT panels with openings, where analytical procedures described in Section 2 can not be applied.

4. Benchmark techno-economic study

The application of low-grade timber in central layers of CLT panels is a well-known concept. However, techno-economic and cost analysis data are very poorly represented in the available literature. Therefore, the data provided in this section could serve as a benchmark for further investigations of hybrid CLT panels in bending. The study considers simply-supported CLT panels:

- standard ones, with equal timber quality (C24) in all layers
- hybrid panels with central layers made of a lower-strength timber (C16).

The study includes 25 CLT panels: 11 standard ones and 14 hybrid ones, as shown in Table 2. The span-to-thickness ratios (L/h) are selected so as to cover a wide range of possible practical applications. The layer thickness is 3cm for all analysed CLT panels. For all CLT panels, the fibre direction of the outside layers is parallel to the span L, while transverse layers are parallel to the B direction (Figures 6 and 7). The square panels are simply supported along edges parallel to B. Elastic material properties of C16 are given in Table 1.

The influence of the following parameters was investigated numerically: number of layers, panel dimensions, timber class in central layers, number of central layers with lower timber class, and the shape and boundary conditions of CLT panels. The panels were analysed for the action of permanent load (G = 3kN/m² – self-weight, the weight of the floating floor and associated CLT walls) and imposed load (Q = 2 kN/m² – for residential buildings, according to [26]), see Figures 6 and 7. Normal and shear stresses were calculated for the ultimate load using partial safety factors γ_G = 1.35 and γ_Q = 1.50.

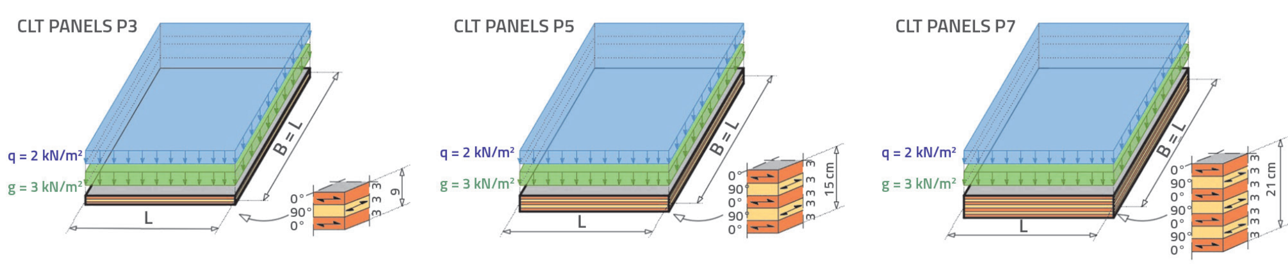

Figure 6. Layout of square CLT panels considered in the study (corrected Figure)

Table 2. Considered lamination schemes and spans of analysed CLT panels

Mark	Description	Lamination scheme
P3-200 P3-250 P3-300	Standard square 3-layer CLT panel, L = B = 2.0 m Standard square 3-layer CLT panel, L = B = 2.5 m Standard square 3-layer CLT panel, L = B = 3.0 m	24/24/24
P3-200-H1 P3-250-H1 P3-300-H1	Hybrid square 3-layer CLT panel, L = B = 2.0 m Hybrid square 3-layer CLT panel, L = B = 2.5 m Hybrid square 3-layer CLT panel, L = B = 3.0 m	24/**16**/24
P5-350 P5-400 P5-450	Standard square 5-layer CLT panel, L = B = 3.5 m Standard square 5-layer CLT panel, L = B = 4.0 m Standard square 5-layer CLT panel, L = B = 4.5 m	24/24/24/24/24
P5-350-H1 P5-400-H1 P5-450-H1	Square 5-layer CLT panel, L = B = 3.5 m, with one C16 layer Square 5-layer CLT panel, L = B = 4.0 m, with one C16 layer Square 5-layer CLT panel, L = B = 4.5 m, with one C16 layer	24/24/**16**/24/24
P5-350-H3 P5-400-H3 P5-450-H3	Square 5-layer CLT panel, L = B = 3.5 m, with three C16 layers Square 5-layer CLT panel, L = B = 4.0 m, with three C16 layers Square 5-layer CLT panel, L = B = 4.5 m, with three C16 layers	24/**16**/**16**/**16**/24
G5-350 O5-350	Standard 5-layer CLT panel with edge opening, L = 3.5 m, B = 6.5 m Standard 5-layer CLT panel with central opening, L = 3.5 m, B = 6.5 m	24/24/24/24/24
G5-350-H3 O5-350-H3	5-layer CLT panel with edge opening and three C16 layers, L = 3.5 m, B = 7.5 m 5-layer CLT panel with central opening and three C16 layers, L = 3.5 m, B = 7.5 m	24/**16**/**16**/**16**/24
P7-500 P7-550 P7-600	Standard square 7-layer CLT panel, L = B = 5.0 m Standard square 7-layer CLT panel, L = B = 5.5 m Standard square 7-layer CLT panel, L = B = 6.0 m	24/24/24/24/24/24/24
P7-500-H3 P7-550-H3 P7-600-H3	Square 7-layer CLT panel, L = B = 5.0 m, with three C16 layers Square 7-layer CLT panel, L = B = 5.5 m, with three C16 layers Square 7-layer CLT panel, L = B = 6.0 m, with three C16 layers	24/24/24/**16**/**16**/**16**/24/24

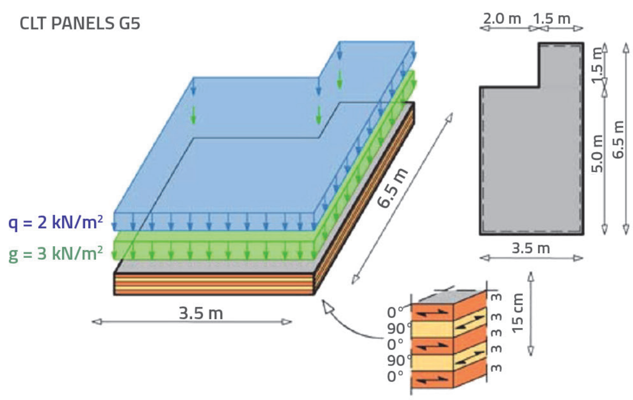

Figure 7. Layout of complex-shape CLT panels considered in the study

The analysis of CLT panels with 3 and 5 layers was made on the basis of γ-method, while CLT panels with 7 layers were calculated according to the Extended Gamma Method. Complex-shape CLT panels (G5 and O5) were analysed numerically using the FLWT-based model. This made available stress distributions for both longitudinal and transverse layers, unlike the analytical procedures. The numerical solution was used for two reasons:

- inability of the analytical methods to account for the 2-way load carrying capacity of CLT panels
- the lack of capability of analytical methods to consider complex plate geometry.

In the finite element model, boundary conditions were prescribed in edge nodes: $U^j = W^j = 0$ for edges parallel to *L*, and $V^j = W^j = 0$ for edges parallel to *B*. The element size was 0.25m, which resulted in 316 elements for G5 and 372 elements for O5. Q8 finite elements with reduced integration were used. The laminas were modelled as a single numerical layer, adopting the linear distribution of displacements across the lamina thickness. The characteristic and design timber strengths of C24 and C16 timber classes are given in Table 3.

The serviceability limit state was taken into account by calculating the final deflection in the centre of the CLT panels.

The partial safety factor for material property γ_m = 1.25, and the modification factor taking into account the effect of the load duration and service class k_{mod} = 0.8, were applied.

Table 3. Design and characteristic strength values (N/mm²) for timber classes C24 and C16

Strength	C16	C24
$f_{m.k}$	16.00	24.00
$f_{m.d}$	10.24	15.36
$f_{VR.k}$	0.80	1.10
$f_{VR.d}$	0.512	0.70

4.1. Results and discussion

4.1.1. Square panels

The results for normal and shear stresses, and for deflection of the analysed square CLT plates, calculated using the γ- and EG methods, are given in Table 4. The following values are presented: normal stress in the mid-span, in the outer layers, from the ultimate load ($\sigma_{m,d}$), shear stress above the support in the centre of gravity of the cross-section, from the ultimate load ($\tau_{R,d}$), and final deflection in the mid-span (w_{fin}).
Relative differences (in %) in stress and deflection values, obtained by comparing hybrid panels against the standard ones, are presented in Table 4. The following price ratio of timber class C24 (PC24) to timber class C16 (PC24) is adopted: PC16/PC24 = 0.785, which corresponds to current trends in the Central European market. According to this, price reduction for a raw timber material is provided for hybrid panels in Table 4.
First, it is obvious from Table 4 that normal and shear stress values do not exceed 50 % of the design strength for all analysed panels (given in Table 3), and so the ultimate limit state is not the driving factor in the design. The maximum deflection is lower than the allowable deflection w_d (L/300) for almost all panels. As expected, lower stress values and higher deflections are obtained for standard CLT panels. However, the percentage deviations range from negligible to very small. A more detailed analysis is given below for various panel types.

3-layer panels (P3): The increase in stress and deflection using hybrid models ranges from 0.04 % to 0.28 % (Table 4), which may be considered negligible. The cost of timber for the production of CLT panels is reduced by approximately 7.76 %. The utilization ratios in relation to the limit values were calculated for the P3-300 panel:

P3-300: $\sigma_{m,d} / f_{m,d,C24}$ = 41.24 %, $\tau_{R,d} / f_{VR,d,C24}$ = 26.71 %, w_{fin} / w_d = 121.30 %

P3-300-H1: $\sigma_{m,d} / f_{m,d,C24}$ = 41.25 %, $\tau_{R,d} / f_{VR,d,C24}$ = 36.52 %, w_{fin} / w_d = 121.30 %.

The highest utilization was obtained for deflection, and is about 21 % above the limit value, showing that the L = 3.0 m span is the limit for the 3-layer plate (h = 9 cm) for a given load. The increase in deflection utilization using the hybrid plate model

Table 4. Stress and deflection results for standard and hybrid square CLT panels, with relative differences (in %) and price reduction values

Panel mark	$\sigma_{m,d}$ [N/mm²]	$\tau_{R,d}$ [N/mm²]	w_{fin} [mm]	Price reduction [%]
P3-200 P3-200-H1	2.946 2.947 (0.05 %)	0.140 0.141 (0.28 %)	2.760 2.761 (0.04 %)	 -7.76
P3-250 P3-250-H1	4.472 4.474 (0.04 %)	0.164 0.164 (0.06 %)	6.164 6.167 (0.04 %)	 -7.76
P3-300 P3-300-H1	6.334 6.336 (0.04 %)	0.187 0.187 (0.04 %)	12.13 12.13 (0.04 %)	 -7.76
P5-350 P5-350-H1 P5-350-H3	3.673 3.685 (0.30 %) 3.693 (0.54 %)	0.119 0.119 (0.17 %) 0.119 (0.50 %)	5.693 5.711 (0.30 %) 5.724 (0.54 %)	 -4.49 -14.81
P5-400 P5-400-H1 P5-400-H3	4.822 4.833 (0.23 %) 4.838 (0.33 %)	0.133 0.134 (0.90 %) 0.135 (1.26 %)	9.574 9.591 (0.18 %) 9.608 (0.35 %)	 -4.49 -14.81
P5-450 P5-450-H1 P5-450-H3	6.028 6.061 (0.55 %) 6.066 (0.63 %)	0.146 0.147 (0.82 %) 0.148 (1.62 %)	14.94 15.03 (0.55 %) 15.04 (0.62 %)	 -4.49 -14.81
P7-500 P7-500-H3	4.214 4.275 (1.43 %)	0.127 0.130 (2.16 %)	9.428 9.643 (2.22 %)	 -10.15
P7-550 P7-550-H3	5.087 5.158 (1.37 %)	0.138 0.141 (2.06 %)	13.60 13.88 (2.05 %)	 -10.15
P7-600 P7-600-H3	6.043 6.124 (1.32 %)	0.148 0.151 (1.92 %)	19.04 19.42 (1.99 %)	 -10.15

Table 5. Stress and deflection results for standard and hybrid complex-shape CLT panels, with relative differences (in %) and price reduction values

Panel mark	$\sigma_{m,d}$ [N/mm²]	$\tau_{R,d}$ [N/mm²]	w_{fin} [mm]	**Price reduction** [%]
G5-350 G5-350-H3	5.50 5.82 (5.50 %)	0.20 0.18 (-10.00 %)	5.92 6.25 (5.28 %)	-14.81
O5-350 O5-350-H3	12.74 12.82 (0.62 %)	0.45 0.39 (-13.33 %)	18.26 19.25 (5.14 %)	-14.81

was negligible. Higher increase in utilization occured for shear stresses (9.81 %).

5-layer panels (P5): The utilization ratios in relation to the limit values were calculated for the P5-450 panel. Again, a driving criterion for these panels is deflection with an obvious utilization of about 100 %. The following values and utilization ratios were obtained:

P5-450: $\sigma_{m,d}$ / $f_{m,d,C24}$ = 39.24 %, $\tau_{R,d}$ / $f_{VR,d,C24}$ = 20.86 %, w_{fin} / w_d = 99.60 %

P5-450-H1: $\sigma_{m,d}$ / $f_{m,d,C24}$ = 39.46 %, $\tau_{R,d}$ / $f_{VR,d,C16}$ = 28.72 %, w_{fin} / w_d = 100.20 %

P5-450-H3: $\sigma_{m,d}$ / $f_{m,d,C24}$ = 39.49 %, $\tau_{R,d}$ / $f_{VR,d,C16}$ = 28.91 %, w_{fin} / w_d = 100.27 %.

For the P5-450-H1 panel (see Table 2 for details), the stress-deflection change ranges from 0.17 % to 0.90 %, while the price reduction is approximately 4.50 %, which is not negligible. For the P5-450-H3 panel, the changes in the stress-deflection state range from 0.35 % to 1.62 %, while the price reduction is about 15 %, which are considerable savings. The analysis shows that the L = 4.5m span is the limit for the 5-layer plate (h = 15cm) for a given load. Obviously, normal and shear stresses are below 40 % utilization and are not relevant.

7-layer panels (P7): The increase in stress-deflection ranges from 1.32 % to 2.22 % (Table 4), while the price reduction is approximately 10 %. For a span of L = 6.0m, the deflection utilization is quite high (between 95 % and 97 %). The use of hybrid panels does not increase the final deflection significantly. The following utilization ratios were obtained:

P7-600: $\sigma_{m,d}$ / $f_{m,d,C24}$ = 39,34 %, $\tau_{R,d}$ / $f_{VR,d,C24}$ = 21,20 %, w_{fin} / w_d = 95,18 %,

P7-600-H3: $\sigma_{m,d}$ / $f_{m,d,C24}$ = 39,87 %, $\tau_{R,d}$ / $f_{VR,d,C16}$ = 29,55 %, w_{fin} / w_d = 97,12 %.

Once again, the normal and shear stress values are below the 40 % utilization and are not relevant.

4.1.2. Complex-shape panels

In contrast with the analysis of square panels, when CLT panels with openings are considered, the driving factor for the design could be both stress and deflection criteria. Stress concentrations occur due to the presence of openings, which can make stress criterion the limiting factor when designing the panel. The stress and deflection distribution in panels G5-350 and O5-350 (Table 2) is shown in Figure 8, where the occurrence of stress concentrations around the openings is illustrated.
Obviously, deflection is still the main factor for design in the case of the G5 panel (with edge opening). The utilization ratio w_{fin}/w_d = 53.57 % is higher in comparison with $\sigma_{m,d}/f_{m,d}$ = 37.89 % and $\tau_{R,d}/f_{VR,d}$ = 35.16 %. However, for the O5 panel (with central opening), the stress criterion ($\sigma_{m,d}/f_{m,d}$ = 83.46 %) is the main

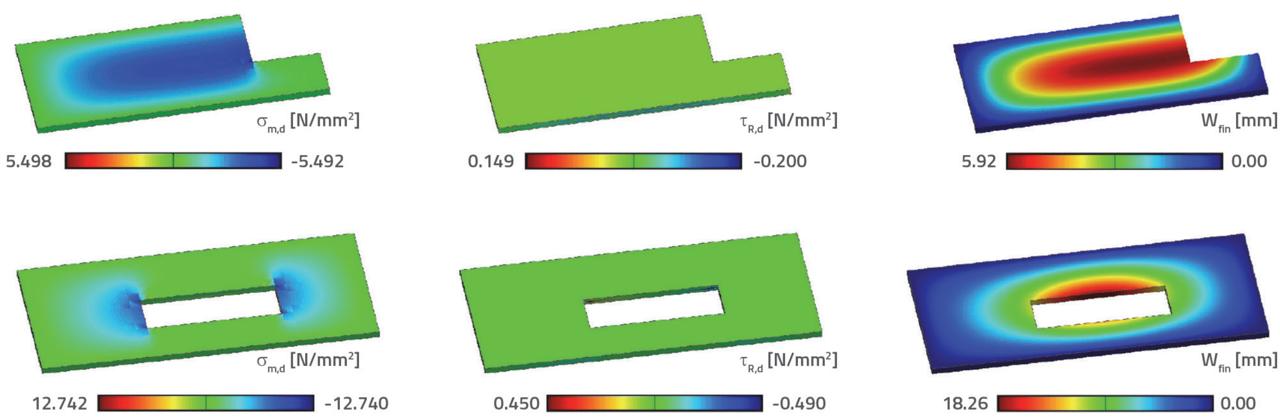

Figure 8. Distribution of stress and deflection in standard complex-shape CLT panels G5-350 (top row) and O5-350 (bottom row) obtained using FLWTFEM software

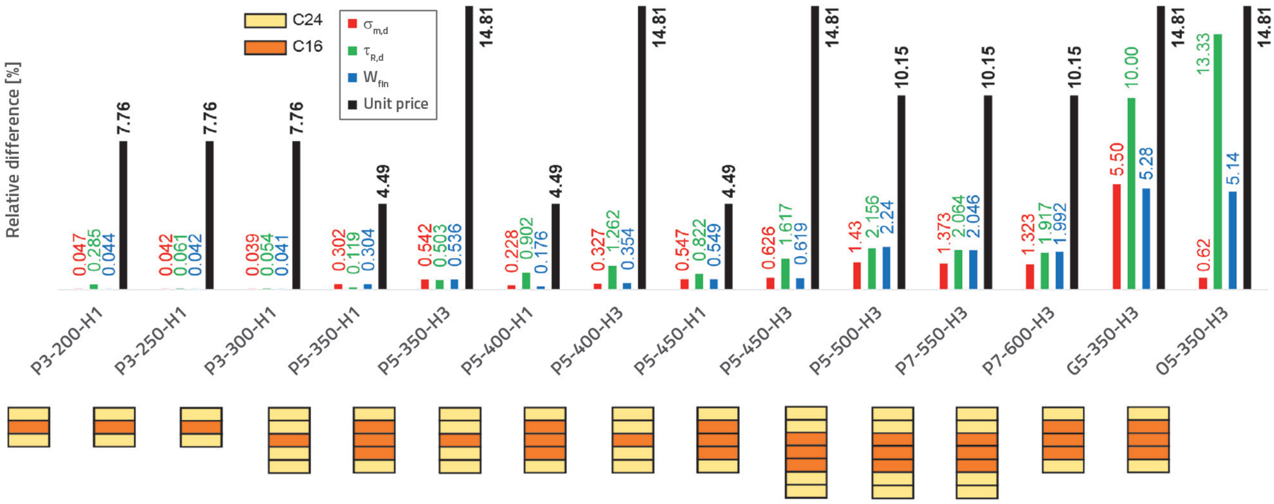

Figure 9. Relative differences in normal stresses ($\sigma_{m,d}$), shear stresses ($\tau_{R,d}$), final deflection (w_{fin}) and unit price for hybrid CLT panels, considering different number/orientation of layers and spans

factor for design. For both types of CLT panels, the relative cost of the raw timber material for the production of CLT panels is lower by 15 % when hybrid panels are used, which confirms validity of the considered concept.

Relative differences in normal and shear stress, final deflection, and unit price, are shown in Figure 9 for all considered hybrid CLT panels. Obviously, the relative cost saving of the raw timber material for the production of CLT panels (black bars in Figure 9) is higher than the % difference in stress and deflection values (coloured bars in Figure 9), confirming the potential of the presented approach.

5.- Conclusions

The paper presents a design approach for the cost-effective hybrid CLT panels in bending, composed of timber layers of dissimilar quality, by applying the existing analytical and novel finite element procedures. A brief overview of analytical procedures, as well as the description of the FLWT-based numerical model of CLT panels implemented in the FLWTFEM original computer code, is presented. A parametric study was performed to compare the stress-deformation state of the selected types of standard and hybrid CLT panels, in order to evaluate the possibilities, advantages or disadvantages of implementation of the presented concept. A benchmark techno-economic study included 25 CLT panels (11 standard and 14 hybrid panels), with 3, 5, and 7 layers, considering different spans, geometries and boundary conditions. The following conclusions can be derived from the above analysis:

- The concept of applying lower timber class in central zones of CLT panels in bending has a visible economic potential that should be considered during the design and production of CLT panels. In addition, the utilisation of hybrid panels implies the use of low strength wood, which can lead to significant financial, logistical and environmental benefits.
- For square simply supported CLT panels with 2 parallel edges, an increase in stress does not exceed 2.2 % when lower grade timber is used in central layers, while the raw material cost-efficiency in production ranges from 4.5 % to as much as 14.8 %.
- Expectedly, final deflection is a driving criterion for the design of all analysed square CLT panels. The percentage of increase in deflection utilization using hybrid CLT panels does not exceed 2.0 %, while the financial savings that can be obtained in the production of CLT panels (ranging from approximately 4.5 % to 14.8 %) are non-negligible.
- Due to stress concentration around the openings, the stress and deflection criteria could be the driving factor for the design of CLT panels with openings. This requires the use of sophisticated numerical methods when designing complex-shape panels so that complex stress state in cut-out zones can accurately be predicted.

Finally, the use of supports along all edges of CLT panels leads to better utilization of CLT material properties, because the 2-way load carrying mechanism is thus activated.

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