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Reliability and punching shear resistance of slabs in non linear domain

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Original scientific paper

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Reliability and punching shear resistance of slabs in non linear domain

In this paper, the punching shear resistance of slabs is numerically evaluated and compared to predictions specified in some design codes. The interaction of major parameters that affect the punching shear behaviour and failure mode of slabs is also studied. Reliability analysis results are presented in terms of the reliability index for various levels of P_u/P_{flex}. In addition to the reliability analysis, the sensitivity analysis is carried out in terms of different load levels so as to study the effect of main variables on the results.

Key words:

reliability analysis, reinforced-concrete slabs, sensitivity analysis, response surface, punching shear resistance

Izvorni znanstveni rad

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Ocjena pouzdanosti i otpornosti ploča na proboj

U radu se numerički ocjenjuje otpornost ploča na proboj te se prikazuje usporedba s predviđanjima utemeljenima na nekim propisima za projektiranje. Analizira se i interakcija glavnih parametara koji utječu na proboj i vrstu sloma ploča. Rezultati analize pouzdanosti prikazani su pomoću indeksa pouzdanosti za razne razine P_u/P_{flex}. Uz analizu pouzdanosti, provedena je i analiza osjetljivosti u smislu različitih razina opterećenja kako bi se na taj način istražio utjecaj osnovnih varijabli na rezultate.

Ključne riječi:

analiza pouzdanosti, armiranobetonske ploče, analiza osjetljivosti, površina odziva, otpornost na proboj posmikom

Wissenschaftlicher Originalbeitrag

Kernou Nassim, Youcef Bouafia, Belakhdar Khalil

Beurteilung der Zuverlässigkeit und des Durchstanzwiderstandes von Platten

In dieser Arbeit wird der Durchstanzwiderstand von Platten numerisch beurteilt und ein Vergleich mit entsprechenden auf Normen für den Entwurf beruhenden Werten aufgestellt. Außerdem wird die Interaktion der wichtigsten Parameter, die das Durchstanzen und die Versagensmechanismen von Platten beeinflussen, analysiert. Resultate der Zuverlässigkeitsanalyse werden mit Hilfe des Zuverlässigkeitsindex für verschiedene Stufen P_u/P_{flex} dargestellt. Außer der Zuverlässigkeitsanalyse wird auch eine Sensibilitätsanalyse für einzelne Laststufen durchgeführt, um den Einfluss der Hauptvariablen auf die Resultate zu untersuchen.

Schlüsselwörter:

Zuverlässigkeitsanalyse, Stahlbetonplatten, Sensibilitätsanalyse, Antwortfläche, Durchstanzwiderstand

1. Introduction

The failure modes of reinforced concrete slabs directly supported by columns can be categorized as: flexural, punching shear, and wide beam failure. Generally, the ultimate strength of such slabs is often determined by the punching shear failure load, which is generally smaller than the flexural failure load. The punching shear mechanism can be described as a sudden shear failure that occurs at the column-slab connection. In other words, a small portion of the slab is punched out from the main slab whereas the rest of the slab remains rigid. Thus, the structure is considered totally collapsed since after punching the connection could lose its shear and bending capacity. Most research on the punching shear strength of slabs has been concerned with the generation of experimental data on simply supported slabs and the development of empirical equations. A few analytical analyses have also been proposed by various investigators based on different models. The best known empirical formulas for predicting the shear strength of slabs as a result of experimental and analytical studies are those suggested by Staller [1], Salim and Sebastian [2], and the ACI-318 code approach [3]. However, In order to ensure compliance with the serviceability requirement of slabs, it is necessary to predict the cracking and deflections of slab structures under service loads. In order to assess the margin of safety of slab structures against failure, an accurate estimation of the ultimate load is also essential, in addition to the prediction of the load-deformation behaviour of the slab throughout the elastic and inelastic response range. Marzouk and Hussein [4]investigated experimentally the behaviour of high-strength concrete slabs. Seventeen reinforced concrete slabs were tested to investigate strength characteristics of high strength concrete slabs subjected to punching shear failure. Kuang and Morley [5] studied the punching shear behaviour of twelve restrained reinforced concrete slabs, which were supported and restrained at all four edges. They investigated the effect of the degree of edge restraint, reinforcement ratio, and span to depth ratio, on the structural behaviour and the punching shear capacity of slabs. They observed that the punching shear capacity is much higher compared to predictions given according to the Johnson's yield-line theory, BS-8110 [6], and ACI-318 [3]. They concluded that the enhanced punching shear capacity was a result of the compressive membrane action caused by restraining action at slab boundaries. Tomaszewicz [7] tested 19 flat square slabs made of high strength concrete with orthogonal, equally spaced, flexural reinforcement, and without shear reinforcement. Slabs were supported along the edges and loaded at mid-span by a concentrated load to failure in punching. Gardner and Shao [8] presented an experimental work on the punching shear of two-bay-by-two-bay flat reinforced concrete slabs; the results showed that the interior column slab connection is more critical than the edge and corner column connections in properly designed multi-bay flat slabs. Osman et al. [9] conducted an experimental work to investigate the behaviour of high-strength lightweight concrete slabs under punching load. The objective and scope of this research is to study the punching shear behaviour, and the major material and geometrical parameters that strongly affect such behaviour.

In real conditions, the complex behaviour of concrete slabs leads to strong uncertainties related to material properties of slabs and loading conditions. These uncertainties must be taken into account in design guidelines for such structures, especially when a structure does not fit exactly into any standard due to its size, new material properties, and complexity or multidisciplinary nature. In these cases, probabilistic analysis can be pursued since design standards are unable to cover the full range of applications that engineers are able to conceive. In the context of reinforced concrete structures, an interesting strategy is to use the response surface approach to evaluate the reliability analysis. This strategy is now widely used in cases when the response of the surface is approximated by an analytical function and the reliability analysis problem is solved for this approximation.

The reliability analysis of a reinforced concrete slab made of HSC (high-strength concrete) and OC (ordinary concrete) is conducted in the paper. The response surface method was used to evaluate an explicit second order polynomial performance function. The random input parameters are: concrete strength, steel yield stress, reinforcement ratio, and the applied load. Failure is assumed to occur when displacements exceed a prescribed limit. Two cases were analysed: reliability analysis using the response surface method and Monte Carlo simulation, and the sensitivity analysis.

Considered as one of the most widely used approximation methods, the response surface method (RSM) has been successfully used in many areas [10]. Various RSM methods have recently been proposed [10, 11]. To achieve these objectives, a reliability-mechanical approach has been developed, which aims to accomplish the coupling between the non-linear finite element of slabs calculations and calculations using response surface to calculate the reliability index.

2. Program for non linear finite element analysis of slabs

A nonlinear finite element code NLFEAS (Non-Linear Finite Element Analysis of Slabs) was used for the nonlinear analysis of concrete slabs. NLFEAS has been developed to predict and study behaviour of the normal and high-strength concrete [12, 13]. The NLEFAS program interface is shown in Figure 1.

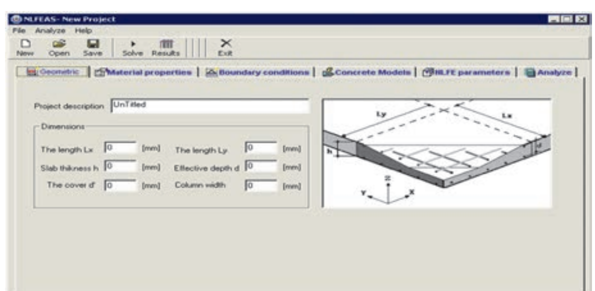

Figure 1. NLEFAS program interface

Table 1. Comparison of predicted and experimental results

Experimental		FE Analysis		FEA/EXP	
Ultimate load P_u [kN]	Ultimate deflection D_u [mm]	Ultimate load P_u [kN]	Ultimate deflection D_u [mm]	P_u / P_u	D_u / D_u
2050	8.52	1920	8.57	0.937	1.006

Table 2. Properties of selected slabs.

References	Slab	Span L [mm]	Depth d [mm]	Column c [mm]	Concrete strength f'_c [MPa]	Steel yielding f_y [MPa]	Steel ratio ρ [%]
Tomaszewicz [7]	ND 65-1-1	2500	300	200	64.3	500	1.37
	ND 95-1-1	2500	300	200	83.7	500	1.37
	ND 95-1-3	2500	300	200	89.9	500	2.29
	ND115-1-1	2500	300	200	112	500	1.37
	ND 65-2-1	2200	220	150	70.2	500	1.56
	ND 95-2-1	2200	220	150	88.2	500	1.56
	ND 95-2-3	2200	220	150	89.5	500	2.33
	ND115-2-1	2200	220	150	119	500	1.56
	ND115-2-3	2200	220	150	108.1	500	2.33
	ND 95-3-1	1100	100	100	85.1	500	1.44
	ND95-2-3D	2200	220	150	80.3	500	2.33
Bani-Yasin [14]	H 1	1500	130	250	72.6	468	0.79
Marzouk & Hussein [4]	HS 3	1500	95	150	69	496	1.47
	HS 8	1500	120	150	69	420	1.11
	HS 9	1500	120	150	74	420	1.61
	HS 10	1500	120	150	80	420	2.33
	HS 13	1500	70	150	68	496	2.00
Elstner [15]	B 09	1828	114.3	254	43.9	341	2.00
	B 14	1828	114.3	254	50.5	325	3.00

2.1. Validation of NLFEM program

Results of the nonlinear finite element analysis of slabs investigated in terms of ultimate load are compared with experimental measurements, as shown in Table 1. Load-deflection curves of slabs selected for the present finite element analysis and experimental results are given in Figure 2.

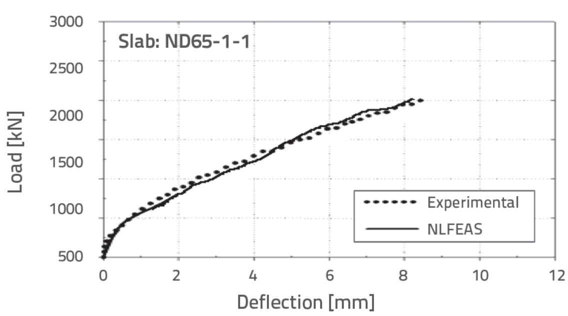

Figure 2. Comparison of predicted and experimental load-deflection curves

It can be seen in Figure 2 and Table 1 that the finite element model's performance is satisfactory, and that it predicts accurately the real behaviour of slabs.

3. Comparison of experimental punching strength predictions

Nineteen slabs were selected from available literature, i.e. from studies conducted by Tomaszewicz [7], Bani-Yasin [14], Marzouk and Hussein [4], and Elstner and Hognistad [15]. It should be noted that all these slabs were subjected to punching shear failure loads. These experimental results were used to check the validity of the punching shear strength formula given in the American concrete institute building code (ACI-318-02) [3], the British standard (BS-8110-85) [6], and the Canadian standard (CSA-A23-94) [16], and to check validity of the computer program presented in the paper. The selected slabs are simply supported square slabs, with a considerable variety of concrete strengths, slab reinforcement ratios and slab depths. They are represented in the various studies as shown in Table 2.

Generally, the ultimate punching shear force for slab strength without shear reinforcement is given by the following equation:

$$V_u = V_c$$
$$V_c = \nu_c \cdot u \cdot d \tag{1}$$

Where is:
V_c - the concrete shear stress
u - the critical section perimeter
d - the slab effective depth.

The punching shear strength equations are reviewed in the following codes. ACI-318-02 [3] identifies the concrete shear strength of slabs as follows:

$$V_c = \min\begin{Bmatrix} (1+\dfrac{2}{\beta_c})\dfrac{\sqrt{f_c'}}{6} b_0 \cdot d \\ (2+\dfrac{\alpha_s \cdot d}{b_0})\dfrac{\sqrt{f_c'}}{12} b_0 \cdot d \\ \dfrac{\sqrt{f_c'}}{3} b_0 \cdot d \end{Bmatrix} \quad [\text{N}] \tag{2}$$

Where is:
β_c - the ratio of the minimum to maximum dimensions of the column sides
b_0 - the critical section perimeter
α_s - the scale factor based on the location of the critical section $\alpha_s = \begin{Bmatrix} 4 \\ 3 \\ 2 \end{Bmatrix}$

4 for interior column, 3 for edge column, and 2 for corner column. The location of critical section (perimeter) suggested by the ACI code is at 0.5 *d* from the column face. According to the Canadian code (CSA-1994) [16], the concrete punching shear strength is taken as the minimum of the following three limits:

$$V_c = \min\begin{Bmatrix} 0,2(1+\dfrac{2}{\beta_c})\sqrt{f_c'} b_0 \cdot d \\ (2+\dfrac{\alpha_s \cdot d}{b_0})\sqrt{f_c'} b_0 \cdot d \\ 0,4 \cdot \sqrt{f_c'} b_0 \cdot d \end{Bmatrix} \quad [\text{N}] \tag{3}$$

Where is:
b_c - the ratio of the minimum to maximum dimensions of the column sides
b_0 - the critical section perimeter
α_s - the scale factor based on the location of the critical section; it is the same as in the ACI code.

The punching shear is checked at the critical section, which is located at the distance of 0,5 d from the face of the column.
The British Standard code (BS-8110) [6] specifies the concrete punching shear strength as follows:

$$V_c = 0,79(100 \cdot \rho)^{\frac{1}{3}}(\frac{400}{d})^{\frac{1}{4}} \cdot b_0 \cdot d \quad [\text{N}] \tag{4}$$

Where is:
f_{cu} - the characteristic cube strength
b_0 - the perimeter of the critical section.

However, the following limitations also apply:

$$\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}} \geq 1$$
$$0,15 \leq (100 \cdot \rho) \leq 3$$
$$\left(\frac{400}{d}\right) \geq 1 \tag{5}$$
$$V_c \leq \min\left(0,8\sqrt{f_{cu}} b_0 \cdot d,\ 5b_0 \cdot d\right)$$

According to BS-8110 [6], the location of critical section is at (1,5.d) from the column face.
In this section, all selected slabs failed in punching shear. Thus in the present finite element analysis the ultimate load (P_u) obtained by the present computer program is considered as the ultimate punching strength (V_u), since the program is capable of predicting the failure mode based on the incorporated technique (as mentioned in the next section).
The comparison of results obtained using the above codes and the present finite element analysis is given in Table 3 and Figure 3 (ignoring the capacity reduction factors, and concrete strength limits, determined by the above mentioned codes.)
The table gives the experimental and predicted punching capacity ratios for the given slabs. The best accuracy is obtained using the present finite element model and the BS-8110 [6] code, while accuracy is lower if the ACI and CAN codes are applied.
Values predicted in the ACI code are sometimes overestimated, as in the case of slabs (HS9, HS10, HS13, B14), but it should be noted here that these slabs have a high reinforcement ratio. However, the ACI code exhibits a conservative prediction of punching load as compared to other codes, but is unsafe for some slabs.
To check the accuracy of codes as a means for predicting the punching shear strength of slabs at high concrete grades, the ratios between the calculated and experimental results are plotted in terms of concrete strength, bearing in mind that the selected slabs have concrete strength ranging from 44 to 119 MPa, as shown in Figure 4.
Figure 4 shows that most of the predicted results are below the safe margin ($V_u/V_{exp} = 1$), for all values of concrete strength. Also, it can be noticed that the results predicted by the ACI code are more conservative compared to others, especially in the case of slabs (HS10, HS13). On the other hand, the present FE model and BS-8110 remain at the best level of safety, with slightly conservative results, for all values of f_c'.
Therefore, all above mentioned codes and the present FE model may be considered applicable for predicting the punching strength of HSC up to 110 MPa.

Table 3. Properties of selected slabs

Analysis / Analyzed slabs			Eksperimental	Program NLFEAS	Design code predictions			Comparison			
References	Slab	f'_c [MPa]	V_{uexp} [KN]	V_u [kN]	ACI V_u [kN]	CAN V_u [kN]	BS-8110 V_u [kN]	Proračun V_u/V_{uexp}	ACI V_u/V_{uexp}	CAN V_u/V_{uexp}	BS-8110 V_u/V_{uexp}
Tomaszewicz [7]	ND 65-1-1	64.3	2050	1920	1603.75	1924.49	1823.43	0.94	0.78	0.94	0.89
	ND 95-1-1	83.7	2250	2340	1892.75	2195.7	1990.95	1.04	0.81	0.98	0.88
	ND 95-1-3	89.9	2400	2580	1896.31	2275.57	2417.44	1.08	0.79	0.95	1.01
	ND115-1-1	112	2450	2540	2116.6	2539.92	2193.94	1.04	0.86	1.04	0.90
	ND 65-2-1	70.2	1200	1120	909.35	1091.22	1142.34	0.93	0.76	0.91	0.95
	ND 95-2-1	88.2	1100	1160	1019.29	1223.15	1232.65	1.05	0.93	1.11	1.12
	ND 95-2-3	89.5	1450	1480	1026.77	1232.13	1416.81	1.02	0.71	0.85	0.98
	ND115-2-1	119	1400	1460	1183.96	1420.75	1362.07	1.04	0.85	1.01	0.97
	ND115-2-3	108.1	1550	1640	1183.96	1354.12	1508.85	1.06	0.73	0.87	0.97
	ND 95-3-1	85.1	330	300	1128.43		324.32	0.91	0.75	0.89	0.98
	ND95-2-3D	80.3	1250	1180	246	1167.08	1366.5	0.94	0.78	0.93	1.09
B-Yasin [14]	H 1	72.6	468	444	972.57	673.47	490.65	0.95	1.20	1.44	1.05
Marzouk & Hussein [4]	HS 3	69	356	340	561.22	309.34	318.97	0.96	0.72	0.87	0.90
	HS 8	69	436	412	257.78	430.62	405.6	0.94	0.82	0.99	0.93
	HS 9	74	543	536	358.85	445.94	469.91	0.99	0.68	0.82	0.87
	HS 10	80	645	724	371.62	463.67	545.64	1.12	0.60	0.72	0.85
	HS 13	68	267	240	169.32	203.19	231.34	0.90	0.63	0.76	0.87
Elstner [15]	B 09	43.9	504.9	456	371.89	446.27	478.87	0.90	0.74	0.88	0.95
	B 14	50.5	578.3	532	398.87	478.65	574.37	0.92	0.69	0.83	0.99
							Aeverage	**0.99**	**0.78**	**0.94**	**0.95**
							Standard deviation	**0.07**	**0.13**	**0.15**	**0.07**

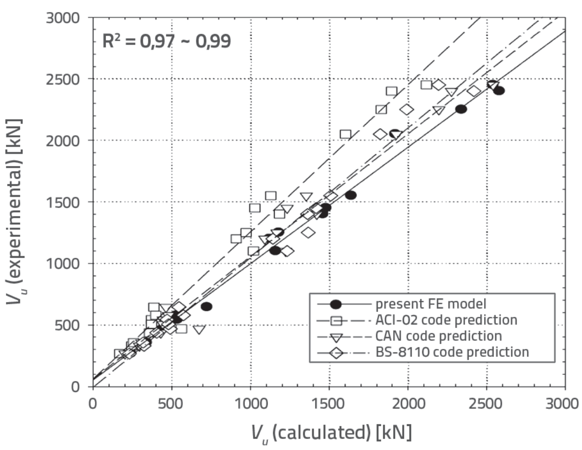

Figure 3. Comparison of experimental and predicted results

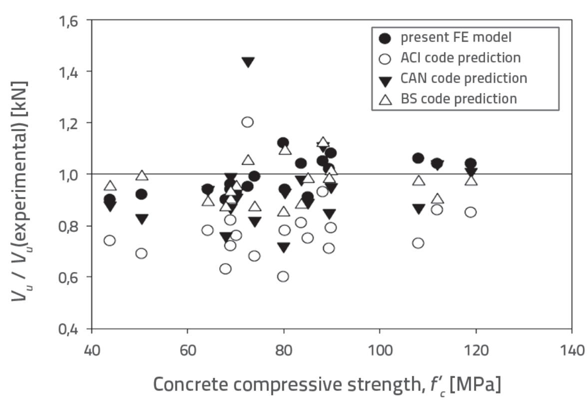

Figure 4. Comparison of experimental and predicted ultimate shear strength in terms of concrete strength

4. Prediction of failure mode

In the analysis of reinforced concrete slabs, it is important to predict their failure mode. Generally, the column punching through the slab is regarded as shear failure, while flexural failure is the failure resulting from rapidly increasing deflection, as defined by Elstner and Hognestad [15] and many other researchers [9, 12, 17]. For this reason, the failure mode may be predicted using the ratio (ϕ_0), which is defined as the ratio of the predicted ultimate load (P_u) to the calculated flexural capacity of the slab (P_{flex}), or:

$$\phi_0 = P_u / P_{flex} \tag{6}$$

This technique was first introduced by Hognestad [15] and has been used by other researchers [4, 5, 18]. In this case the ultimate flexural capacity of the slab (P_{flex}) is estimated using the yield line theory, with the assumption that shear failure will not occur. As a result, for $\phi_0 \leq 1$ the failure mode is governed by shear failure, while for $\phi_0 > 1$ the failure is governed by flexural failure. The failure load required for the flexural mechanism can be determined using the following equation, which is based on the virtual work of the yield line analysis.

$$P_{flex} = 8 \cdot M \cdot \left(\frac{L}{L-c} - 0{,}172 \right) \tag{7}$$

Where:
- L - side dimension of the square slab
- c - side dimension of the square column
- M - ultimate flexural moment per unit width, which may be calculated from the widely accepted equation proposed by the ACI-318 code as follows:

$$M = \rho \cdot f_y \cdot d^2 \left[1 - \frac{k_2}{k_1 k_3} \cdot \frac{\rho \cdot f_y}{f_c^{'}} \right] \tag{8}$$

Where:
- ρ - reinforcement ratio
- f_y - ultimate yield stress of steel
- f'_c - concrete compressive strength
- d - effective slab depth
- k_1, k_2, k_3 - stress block parameters.

In the normal strength concrete, the value of ($k_2 / k_1 k_3$) is frequently taken to be constant and equals to 0.59. However, modified stress block parameters according to the paper proposed by Attard and Stewart [19] are adopted in this study. The following equations are proposed for the evaluation of these parameters for the concrete strength of up to 120 MPa:

$$k_2 = 1{,}0948 (f_c^{'})^{-0{,}091} \geq 0{,}67$$

$$k_1 \cdot k_3 = \begin{cases} 1{,}2932 (f_c^{'})^{-0{,}0992} \geq 0{,}71 \\ 0{,}647 (f_c^{'})^{-0{,}0998} \geq 0{,}58 \end{cases} \tag{9}$$

The above method for predicting the slab failure type has been incorporated into the modified program (NLFES). It should be noted that the parameter $k_1 k_3$ based on dog bone tests, was adopted. The results obtained are compared with the experimental observations, as presented in Table 4. The ultimate load predicted by the computer program is the load that satisfies the failure criteria (reaches the ultimate equivalent strain).

As can clearly be seen, the presented finite element model is capable of predicting the slabs' failure mode with a very good accuracy. Thus, the use of $\phi_0 (P_u / P_{flex})$ to predict the failure mode, as proposed by Hognestad [15], is both practical and reliable. Therefore, when the program indicates that the slab failure occurred due to punching shear, the ultimate load (P_u) result given by the program is considered in the program as the punching shear strength (V_u).

5. Major parameters affecting punching shear behaviour of slabs

Based on parametric studies of geometry and materials, as presented in previous section, it was established that there are parameters of lesser importance that have only a slight effect on slab behaviour, or that affect the ultimate load only, such as the grade of steel, arrangement of reinforcement, and boundary conditions, [20]. On the other hand, there are important parameters that significantly influence the total slab response, such as the concrete strength, reinforcement ratio, and span-to-depth ratio. Hence, analytical investigations were carried out in this section on a supposed full scale slab to study joint influence of these important parameters on slab behaviour. Since it was established that the present FE model has a good capability to predict slab behaviour with a satisfactory accuracy and in close agreement with experimental test, a supposed full scale simply supported square slab with real dimensions was investigated.

Slab dimensions are shown in Figure 5. The slab was analysed by considering different values of major parameters as follows:
- reinforcement ratio (ρ) [0.5; 0.75; 1.00; 1.50; 2.00] %
- concrete strength (f'_c) [30, 50, 110] MPa, and
- span-to-depth ratio (s/d) using different effective depths (d) [80, 100, 150, 200,250] mm with the single span of 4.0 m.

The analysis was carried out using the developed finite element program (NLFEAS) and the material parameters and models used in previous chapter. The obtained results are described in the following section in terms of ultimate load capacity and failure mode as influenced by the major parameters, including the steel ratio, span-to depth ratio, and concrete strength.

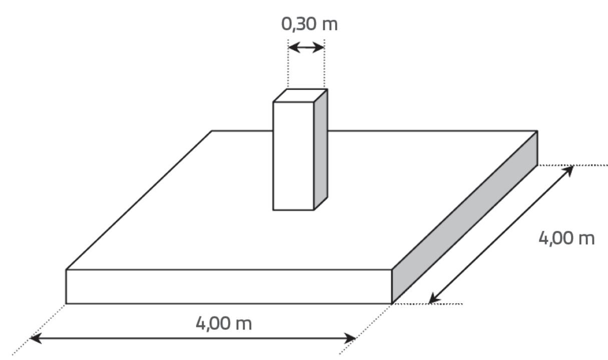

Figure 5. Slab dimensions

Table 4. Comparison of experimental and predicted failure modes

Analyzed slabs / Comparison of results				Experimental		Finite element			
References	**Slab**	f'_c [MPa]	ρ [%]	**Ultimate load P_u** [kN]	**Način sloma**	P_u	P_{flex}	P_u/P_{flex}	**Corresp. failure mode**
Tomaszewicz [7]	ND 65-1-1	64.3	1.37	2050	Punching	1920	4465.12	0.43	Punching
	ND 95-1-1	83.7	1.37	2250	Punching	2340	4178.57	0.56	Punching
	ND 95-1-3	89.9	2.29	2400	Punching	2580	6615.38	0.39	Punching
	ND115-1-1	112	1.37	2450	Punching	2540	4233.33	0.60	Punching
	ND 65-2-1	70.2	1.56	1200	Punching	1120	2434.78	0.46	Punching
	ND 95-2-1	88.2	1.56	1100	Punching	1160	2468.09	0.47	Punching
	ND 95-2-3	89.5	2.33	1450	Punching	1480	3609.76	0.41	Punching
	ND115-2-1	119	1.56	1400	Punching	1460	2561.40	0.57	Punching
	ND115-2-3	108.1	2.33	1550	Punching	1640	3727.27	0.44	Punching
	ND 95-3-1	85.1	1.44	330	Punching	300	491.80	0.61	Punching
	ND95-2-3D	80.3	2.33	1250	Punching	1180	3575.76	0.33	Punching
B-Yasin [14]	H 1	72.6	0.79	468	Flex-Punching	444	493.33	0.90	Punching
Marzouk & Hussein [4]	HS 3	69	1.47	356	Flex-Punching	340	478.87	0.71	Punching
	HS 8	69	1.11	436	Punching	412	490.48	0.84	Punching
	HS 9	74	1.61	543	Punching	536	687.18	0.78	Punching
	HS 10	80	2.33	645	Punching	724	1019.72	0.71	Punching
	HS 13	68	2.00	267	Punching	240	328.77	0.73	Punching
Elstner & Hognestad [15]	B 01	14.2	0.50	178.3	Flex	172	160.75	1.07	Flex
	B 02	47.6	0.50	200.1	Flex	194	190.20	1.02	Flex
	B 04	47.7	1.00	333.6	Flex	300	297.03	1.01	Flex
	B 09	43.9	2.00	504.9	Punching	456	608.00	0.75	Punching
	B 14	50.5	3.00	578.3	Punching	532	844.44	0.63	Punching
Taylor	S1*	35	0.6	32	Flex	32.08	13.42	2.39	Flex
	S7*	38	0.8	31	Flex	30.52	11.74	2.60	Flex
*Slabs subject to uniform load									

5.1. Effect of major parameters on ultimate load

It can be seen in Figure 6 that the ultimate load capacity differs in each direction and level, depending on the steel ratio, span-to-depth ratio, and concrete strength. At low value of ρ (0.50 %), the ultimate load P_u increases to 670 % and 828 % when increasing the slab effective depth *d* from 80 to 250 mm, for f'_c of 30 and 110 MPa, respectively. At high value of ρ (2.00 %), P_u increases to the corresponding percentages of 485 % and 707 %, for f'_c = 30 and 110 MPa, respectively, for the same change in d. It was established that the influence of concrete grade on the ultimate load is small at low reinforcement ratio (ρ = 0.5 %) and low slab thickness (*d* = 80 mm), while the load carrying capacity increases by 14.7 % when the concrete strength is changed from 30 to 110 MPa. However, at a high value of ρ (2.0 %), and with d =

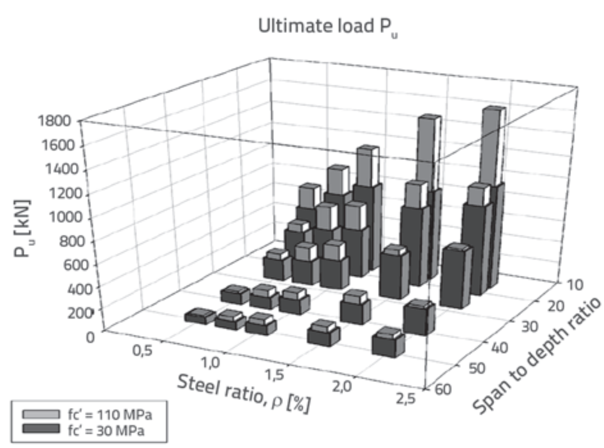

Figure 6. Effect of concrete strength on ultimate capacity of slab

250 mm, the ultimate load increases to 70 %, for the same change in concrete strength. Hence, the improvement in slab capacity by increasing the concrete strength gives best results in slabs with a high reinforcement ratio and high slab thickness (low span-to-depth ratio). However, it is useless to augment the concrete strength using high strength concrete in the case of slabs with a low reinforcement ratio and low thickness.

5.2. Effect of major parameters on failure type

The above method is used to identify the failure type, and the results of the present finite element analysis are plotted in Figures (7, 8 and 9) for f'_c = 30, 50, 100 MPa, respectively. These figures reveal that the punching shear failure mode is expected for slabs with a low span-to-depth ratio and high reinforcement ratio, for all concrete strength levels, and for the low strength concrete in particular. It was furthermore found that the type of failure is ductile flexural at a low reinforcement ratio ρ < 1 % for all slabs made of high strength concrete.

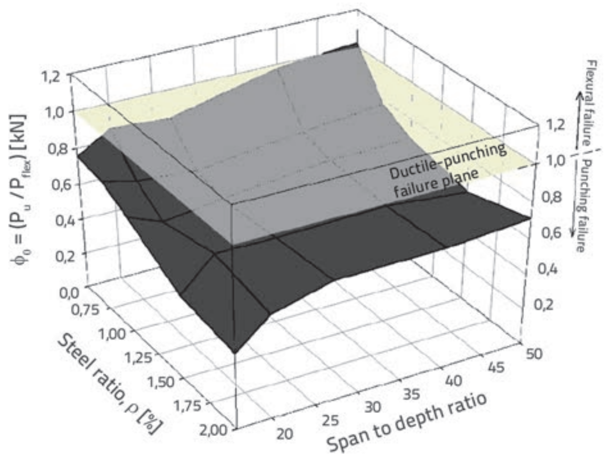

Figure 7. Failure type prediction for f'_c = 30 MPa

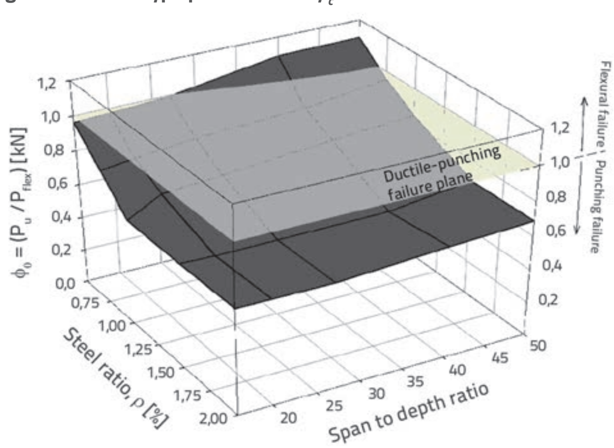

Figure 8. Failure type prediction for f'_c = 50 MPa

The effective depth or span-to-depth ratios have a small influence on failure type at a high reinforcement ratio. However, such ratios may influence the failure type at a low reinforcement ratio, especially for the low and normal strength concrete. Besides, it can be seen in Figures (7-9) that in case of f'_c =30, 50 and 110 MPa, the shear failure mode is obtained at ρ > 0.3 %, 0.6 %, and 1.00 %, respectively. It can therefore be concluded that slabs made of the low and normal strength concrete are generally more vulnerable to punching shear compared to those made with the high strength concrete, especially at high reinforcement ratios.

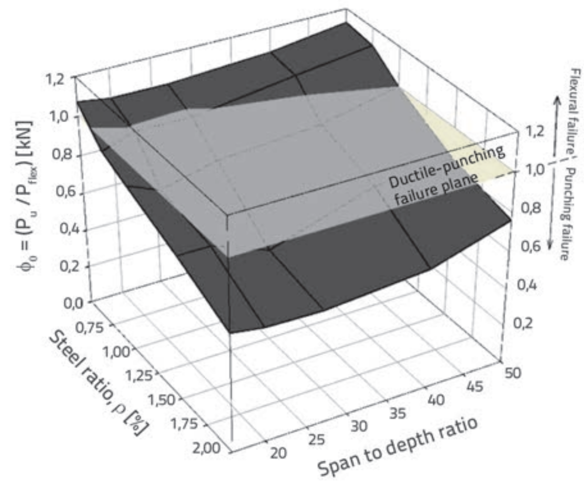

Figure 9. Failure type prediction for f'_c = 110 MPa

6. Slab reliability study

6.1. Performance function

The performance function, also known as the limit state, separates the data space into two regions: the safe and failure region. The simulation results enable development of an artificial performance function (response surface), which approximately describes the collapse prevention state of the slabs. This is performed by means of a polynomial regression over a set of results that are established according to a full factorial table. In our case, a quadratic regression is used to interpolate the obtained maximum deflection where several levels are considered for each factor: concrete resistance, steel yield stress, effective depth, and steel ratio.

Finite element reliability methods are characterized by response quantities from a finite element solution entering the performance function. For instance, a simple threshold performance function is:

G = hreshold - response quantity.

Considering the limit state of deflection, the failure function is written as follows (10):

$$G(f'_c, f_y, d, \rho) = (L/500) - g(f'_c, f_y, d, \rho) \quad (10)$$

where:

L/500 - defines critical deflection according to EC 02 [21].

The identified response surfaces corresponding respectively to high strength concrete slabs (HSC) and ordinary concrete (OC) can readily be obtained from Table 6 as follows. A summary of random variables, with their probability distribution and parameters, is shown in Table 5.

$$
\begin{aligned}
g(f'_c, f_y, \rho, d) = (&-0{,}00122500120038601 \cdot f'^{2}_c + 0{,}00267246465142617 \cdot f'_c \cdot f_y - 0{,}0284541275397354 \cdot f'_c \cdot \rho - 0{,}00239308507731399 \cdot f'_c \cdot d - 0{,}402347866577491 \cdot f'_c \\
&- 0{,}00160494157962584 \cdot f_y^2 + 0{,}00306486937542444 \cdot f_y \cdot \rho + 0{,}0115083956160873 \cdot f_y \cdot d + 0{,}896029536088040 \cdot f_y + 1{,}062341704809601 \cdot \rho^2 \\
&+ 0{,}0484911191408620 \cdot d \cdot \rho - 16{,}0222314506990 \cdot \rho - 0{,}00199483323484406 \cdot d^2 + 0{,}374175860582561 \cdot d - 186{,}471678670218)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
g(f'_c, f_y, \rho, d) = (&-0{,}00807625426347610 \cdot f'^{2}_c - 0{,}00656764689271723 \cdot f'_c \cdot f_y + 0{,}0277941806483146 \cdot f'_c \cdot \rho + 0{,}00753561317629636 \cdot f'_c \cdot d + 0{,}5908996314087630 f'_c \\
&- 0{,}00208879872667190 \cdot f_y^2 + 0{,}14063829984904 \cdot f_y \cdot \rho - 0{,}00252944665820754 \cdot f_y \cdot d + 2{,}14914137789604 \cdot f_y + 2{,}15727734249363 \cdot \rho^2 \\
&- 0{,}0740173083144320 \cdot d \cdot \rho - 51{,}1372641994972 \cdot \rho + 0{,}00172138023115786 \cdot d^2 + 0{,}196570690049373 \cdot d - 410{,}91915000937)
\end{aligned}
\tag{12}
$$

Table 5. Probabilistic models of random variables

Variable	Distribution	Mean value	COV	Reference
f'_c	Normal	27.5 (OC) / 62.5 to (HSC)	0.11	[22]
f_y	Normal	405	0.08	[22]
ρ	Normal	1.75	0.024	[23]
d	Normal	205	0.03	[24]

Table 6. Reliability study results

Simulation	f'_c [MPa]		f_y [MPa]	ρ [%]	d [mm]	OC				HSC			
	OC	HSC											
	f'_c [MPa]	f'_c [MPa]				D [mm]	P_u / P_f	β *RSM*	β *MCS*	D [mm]	P_u / P_f	β *RSM*	β *MCS*
S1	25	60	400	1.5	200	3.533	0.266	3.12	3.14	2.587	0.231	3.78	3.79
S2	25.5	60.5	410	1.8	210	2.530	0.208	3.64	3.71	1.763	0.174	4.26	4.31
S3	25.5	60.5	410	1.75	210	2.598	0.212	3.6	3.64	1.825	0.178	4.22	4.29
S4	26	61	405	2	205	2.725	0.204	3.69	3.85	1.960	0.168	4.35	4.41
S5	26	61	410	1.65	205	2.939	0.230	3.42	3.43	2.142	0.197	4.07	4.05
S6	25.75	60.75	405	1.5	202	3.390	0.257	3.2	3.21	2.489	0.223	3.86	3.88
S7	25.25	60.25	402	1.6	205	3.092	0.240	3.34	3.34	2.208	0.206	4.01	4.00
S8	25	60	407	1.5	204	3.302	0.253	3.27	3.33	2.399	0.218	3.91	3.94
S9	25.25	60.25	406	1.55	203	3.258	0.249	3.31	3.32	2.367	0.215	3.94	3.96
S10	25.75	60.75	403	1.525	200	3.479	0.259	3.18	3.19	2.546	0.226	3.83	3.85
S11	25.5	60.5	401	1.9	201	2.809	0.222	3.5	3.55	2.023	0.185	4.18	4.24
S12	26	61	408	1.7	206	2.828	0.224	3.48	3.49	2.044	0.191	4.11	4.14
S13	27	62	409	1.85	207	2.592	0.206	3.67	3.74	1.838	0.175	4.25	4.3
S14	27	62	405	1.725	208	2.699	0.216	3.57	3.62	1.924	0.186	4.17	4.22
S15	30	65	410	2	210	2.249	0.185	3.87	3.9	1.543	0.159	4.48	4.55
S16	28	63	404	1.575	204	2.997	0.238	3.36	3.38	2.292	0.209	3.98	3.98
S17	29	64	403	1.6	202	3.049	0.238	3.36	3.41	2.337	0.211	3.96	3.95
S18	29	64	407	1.65	205	2.839	0.224	3.48	3.43	2.139	0.197	4.07	4.01
S19	30	65	410	1.8	200	2.864	0.218	3.55	3.58	2.113	0.190	4.14	4.13
S20	30	65	400	1.5	200	3.307	0.255	3.25	3.3	2.522	0.229	3.8	3.81
S21	30	65	402	1.75	202	2.836	0.221	3.51	3.57	2.082	0.195	4.09	4.07
S22	28.5	63.5	400	2	202	2.553	0.205	3.68	3.73	1.898	0.174	4.26	4.31
S23	27.5	62.5	401	1.5	204	3.281	0.249	3.13	3.16	2.388	0.220	3.89	3.93
S24	29.5	64.5	400	1.85	205	2.584	0.208	3.64	3.66	1.834	0.181	4.22	4.28
S25	25	60	402	1.55	201	3.385	0.256	3.23	3.25	2.470	0.221	3.88	3.91

*L_{slabs} = 2.5 m. E = 200000 [MPa], OC - ordinary concrete, HSC - *h*igh strength concrete,
RSM - response surface method, MCS - Monte Carlo simulation, β - reliability index

Permanent loads are G = self-weight of the slab, and variable loads are Q = 5 KN/m^2. Active gravity loads are computed by adopting the following combination P_u=1.35 · G + 1.5 · Q =24.375 · 4= 97.5 KN. The polynomial response function is determined using the central composite design. In this case, $2^n + 2n + 1$ are used. In order to capture more precisely the non-linearity of the true limit state, mixed terms can be included into the following quadratic polynomial.

6.2. Different stages of reliability index computation using response surface method

It is first assumed that the initial centre point is formed of mean values of random variables for the first iteration. The responses are calculated by conducting nonlinear FEM analysis for slabs (NLFEAS) at the experimental sampling points for the response surface model being considered.

A limit state function is thus generated in terms of basic random variables *k*. The reliability index β and the corresponding coordinates of the checking point and direction cosines are obtained, for each random variable, using the expression for the limit state function and FORM.

The coordinates of the new centre point are obtained by linear interpolation. The updating of the centre point location continues until it converges to a predetermined tolerance level. In the final iteration, the information on the most recent centre point is used to formulate the final response surface. The FORM or SORM is then used to calculate the reliability index and the corresponding coordinates of the most probable failure point. The different steps of the procedure are given in the following chart.

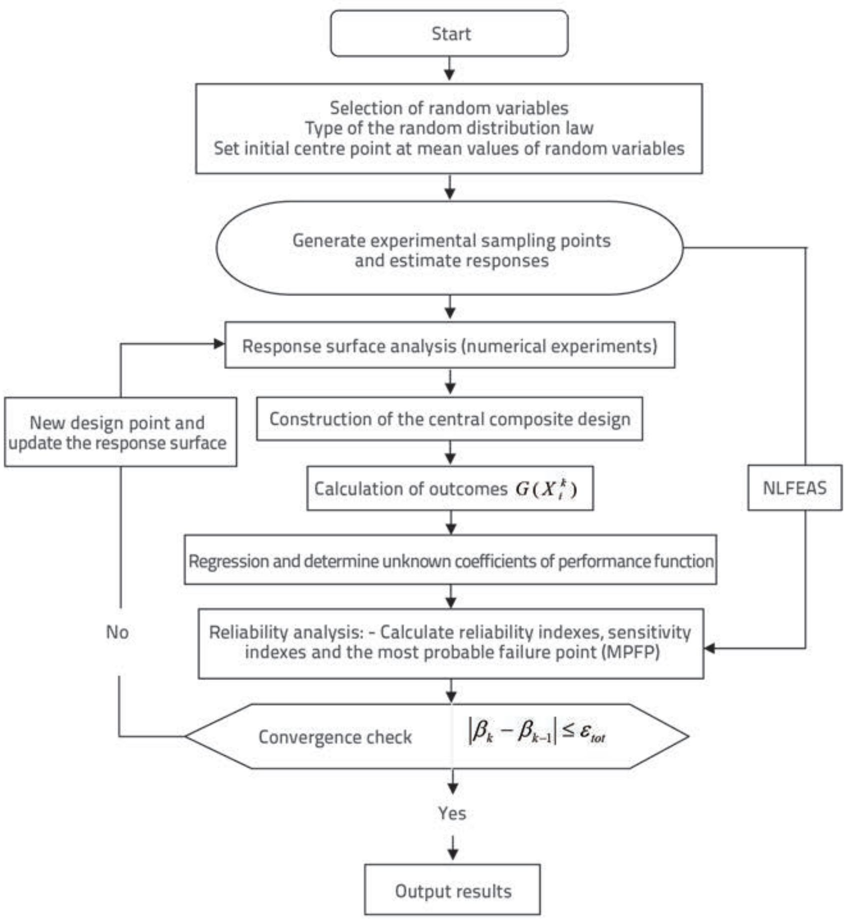

Figure 10. Flowchart of proposed approach

6.3. Case One: Comparison of reliability methods

In this Case One, the aim is to compare the following reliability methods: RSM and Monte Carlo Simulation. The variation of reliability index β in terms of the P_u/P_{flex} for different reliability methods is shown in Figures 11 and 12.

The proposed approach provides valuesof the index of reliability that are similar to the Monte Carlo method. However, the number of required iterations is *Ne* = 1.000.000 (time indicator associated with calculation method), while only 13 iterations are needed in the proposed approach. So the method developed is less time consuming in terms of computation time.

The reliability index is determined for various values of P_u / P_{flex}. Results are listed in Table and are plotted in Figures 11 and 12. The results indicate that the section reliability is highly sensitive to the P_u / P_{flex} ratio and several levels for each factor: concrete resistance, steel yield stress, effective depth and steel ratio. For singly reinforced sections, the reliability index β_{RSM} drops from 4.48 to 3.78 if the P_u / P_{flex} increases from 0.159 to 0.231. Important compressive strength values improve the section reliability, especially for this type of HSC slabs. However, the reliability index β_{RSM} drops from 3.87 to 3.12 when the P_u / P_{flex} is increased from 0.185 to 0.266. The compressive strength improves the section reliability for ordinary concrete slabs, especially when P_u / P_{flex} = 0.222. Nevertheless, the contribution of all parameters is very important for every type of slabs in order to ensure the desired reliability.

6.4. Case Two: Sensitivity analysis

In this Case Two a sensitivity analysis was carried out to study the effect of input parameters on the nonlinear analysis of slabs. Three F values were investigated in order to compare variation of sensitivity in terms of the level of F. The F values are: 97.5 kN, 125 kN, 150 kN. The aim of this analysis is to study the variation of sensitivity when load approaches the yield load of the studied slab. The results

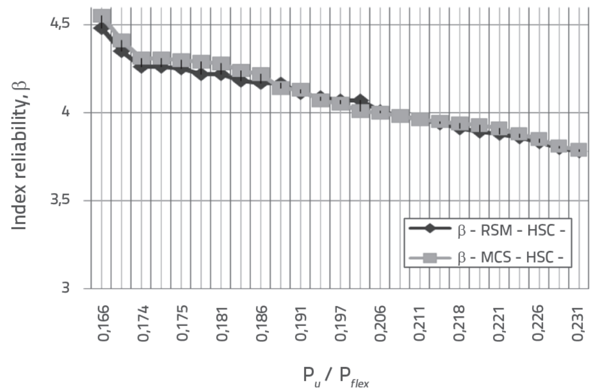

Figure 11. Reliability index of HSC slab versus P_u / P_{flex}

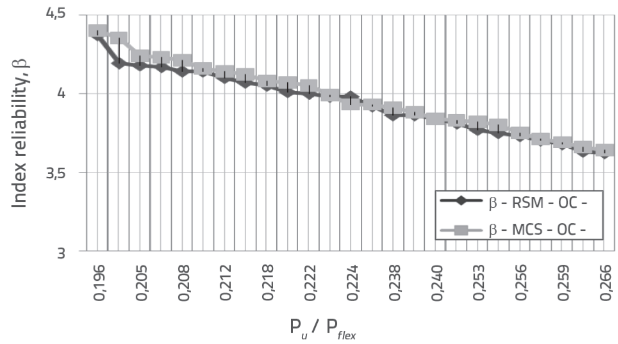

Figure 12. Reliability index of OC slab versus P_u / P_{flex}

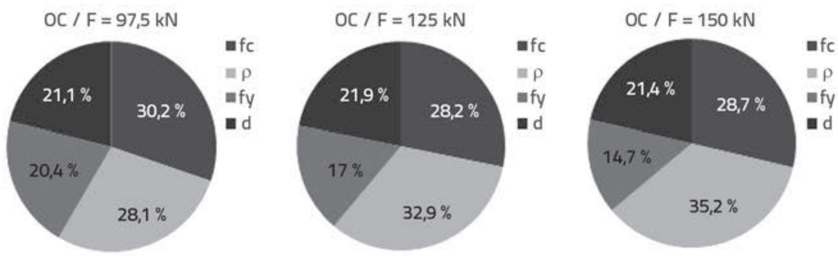

Figure 13. Sensitivity of parameters versus load level variation of OC slabs

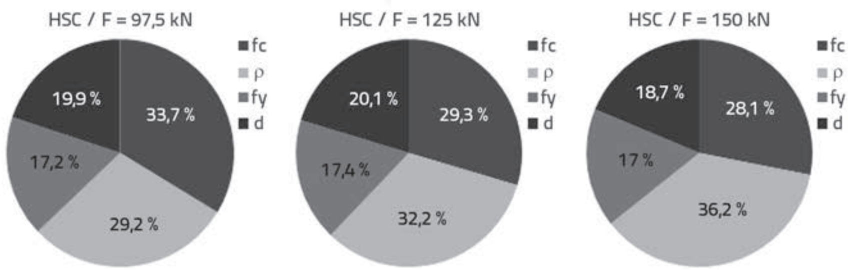

Figure 14. Sensitivity of parameters versus load level variation of HSC slabs

are presented in figures 13 and 14.. Figures shows that the sensibility of input variables varies with the load level.

It can be noted that the sensibility of reinforcement ratio increases as the applied force converges to yielding load (F=150 kN). Thus, at F= 97.5 kN, the sensitivity of ρ was 29.2 % while at F= 150 kN the sensitivity of ρ was 36.2 % for the HSC slab and, at F= 97.5 kN, the sensitivity of ρ was 28.1 %. At F= 150 kN the sensitivity of ρ was 35.2 % for the OC slab. It can be observed that the most important parameter influencing slab behaviour is the reinforcement ratio, which is due to the fact that the strength of such thin structures is strongly dependent on the quantity of reinforcement.

Also, the applied force (F) and concrete strength have a significant effect on slab behaviour, while both steel yield fy and effective depth (d) have a slight impact compared to other parameters.

This variation in sensitivity may occur because, at the slab yielding stage, the reinforcement ratio has a significant effect on slab displacement since at that steel yielding stage the displacement becomes large.

On the other hand, the variation in sensitivity in concrete strength, i.e. the steel yield stress due to load variation, is almost smaller than that of reinforcement ratio, and is not affected by load level variations.

7. Conclusions

The conclusions extracted from the current study are divided into two main categories reflecting the major objectives of the study:

a) Analysis of punching shear resistance of slabs, and
b) Reliability and sensitivity analysis of concrete slabs using RSM.

In addition, the punching shear behaviour, and the most commonly considered parameters that significantly affect the punching shear strength of slabs, are also studied using a real full scale slab. The interaction effect of these parameters is equally studied, and a wide range of information is collected.

The reinforcement ratio may be considered as the first factor that may influence the failure type. Also, the analytical study shows that the shear failure occurs at about ρ > 0.3, 0.6, and 1.00 %, for the concrete strength of *fc′* = 30, 50, and 110 MPa. This means that slabs made of low and normal strength concrete are generally more vulnerable to punching shear than the high strength concrete, especially at high reinforcement ratios.

The reliability analysis of a reinforced concrete slab is presented in the paper. The response surface method is used and an explicit second order polynomial performance function is evaluated. The failure is assumed to occur when displacements exceed the prescribed limit. Two cases of analysis where carried out: reliability analysis using RSM and Monte Carlo Simulation in terms of P_u / P_{flex}, and sensitivity analysis in terms of load level. The following conclusions can be made:

The proposed method works with the response of the physical model through a response surface. The response surface is fully quadratic. Using this technique, the method we propose can reduce the computation cost while ensuring a good accuracy of the results. The Monte Carlo simulation is used to control the validity of results, but can be very time-consuming, which depends on the failure probability level desired.

The sensitivity analysis shows that the most important parameter affecting slab behaviour is the reinforcement ratio. This is quite logical since the strength of such thin structures is strongly dependent on the quantity of reinforcement. Also, the applied force has a significant effect on slab behaviour, while both the concrete strength f'_c and steel yield f_y have a smaller impact compared to other parameters. It was also established that the sensibility of input variables varies depending on the load level, but to a differing extent. It was noticed that the sensibility of reinforcement ratio increases as the applied force converges to yield load (as expected by finite element analysis). This variation in sensitivity may occur because the reinforcement ratio has a significant effect on slab displacement at the slab yielding stage since at that stage, when steel yields, the displacement becomes large.

On the other hand, the variation in sensitivity in concrete strength, i.e. the steel yield stress due to load variation, is almost smaller than that of reinforcement ratio, and is not affected by load level variations.

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